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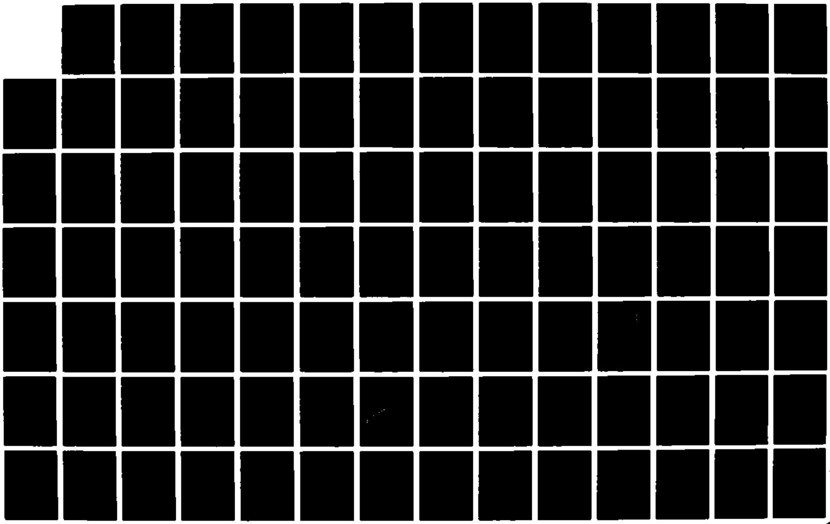

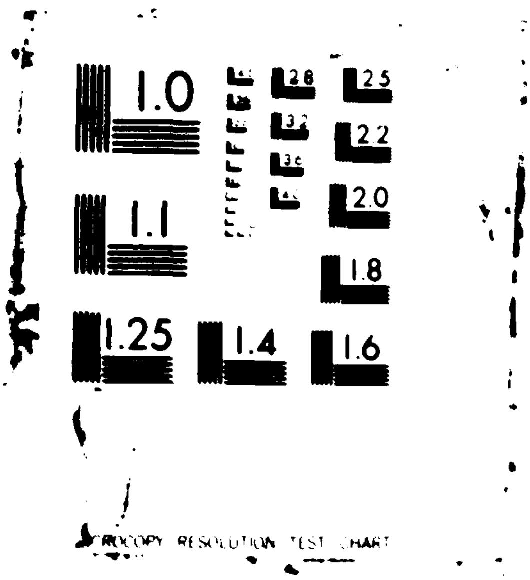
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A method for the prediction of turbulent shear flows with free boundaries is developed. Conditional statistics are used to establish the properties of turbulent and nonturbulent zones. Conditions governing the effect of the fluctuating interface, that separates turbulent and nonturbulent zones, are derived and a closure model for conditioned moments of first and second order are put forward. The resulting second order closure is compared to experiments for several plane shear flows and good agreement is found.

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CONDITIONAL SECOND ORDER CLOSURE FOR TURBULENT SHEAR FLOWS

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Principal
Investigator: W. Kollmann, UC Davis

Summary

The research work during the first year was concentrated on two areas: The foundation of conditional closure schemes in terms of probability density functions (pdf) and the development of a second order closure including intermittency factor and conditional moments. In the theoretical part dynamics of single and multi-point pdf's for velocity and a scalar variable, that can be used for distinction between turbulent and nonturbulent zones, were considered and methods of closure were investigated. The transport of apparent stress in the nonturbulent zone of turbulent shear flows with a free boundary was included in the second order model in terms of their dynamic equations. Conditions governing the effect of the fluctuating interface on mean velocity and apparent stress in both zones were established and closure models were put forward. The resulting second order closure was compared with experiments for several plane shear flows and good agreement was found.

Research Objectives

The objective of the proposed research project is the development of a second order closure model for conditional moments and the intermittency factor. The foundation of the closure scheme are to be investigated and the resulting model should be applicable to a wide range of turbulent shear flows with free boundaries.

Status of research

The research work on this project started in July 1984 with M Mortazavi as a Ph.D. student and S. Byggstoyl fro TU Trondheim (Norway) as a Postdoctoral Fellow.

M. Mortazavi and W. Kollmann worked during the first year on the probability density formulation (pdf) of conditional closures. M. Mortazavi started with a thorough survey of the existing literature on pdf-methods. This lead to the definition of his contribution as the single and multi-point pdf formulation of conditional closure schemes. He then began working on the specific problem of turbulent diffusion of prassive scalars, for which he is currently developing closure models based on the work of Lundgren [1], [2], Ievlev [3], Kuo and O'Brien [4], Kollmann and Janicka [5].

S. Byggstoyl and W. Kollmann developed the second order closure [6] further by considering the transport of apparent stress in the nonturbulent zones of shear flows with free boundaries. It was found that conditional mean velocities and conditional stresses undergo additional transport and production/destruction due to the random fluctuations of the interface separating the turbulent and nonturbulent zones. This effect is represented by additional terms in the transport equations for these moments and it was shown that those terms were linked by a consistency condition and approach particular limit forms as the distance from the turbulent shear layer increases (Corrsin- Kistler relations). These theoretical results are contained in appendix I. The second order closure model was then complemented with the transport equations for the apparent stress in the nonturbulent zone. The condition that the fluctuations in

the nonturbulent zone are irrotational leads to decay laws with distance from the turbulent region, which, together with the consistency condition, restrict the possible closure expressions. The numerical solution of the resulting system of nonlinear parabolic differential equations required simultaneous solution of the stress equations in each zone in order to cope with nonlinear instabilities which frequently occurred in the sequential solution method. Thus a blocksolver was introduced for the stress equations which improved the stability characteristics significiently and lead to a mode rate gain in computing time. The comparison of the results with experiments in several plane shear layers is quite satisfactory. The results are presented in detail in appendix II.

The research on conditional closure schemes lead to a new idea. Multi-scale models based on conditional statistics using multizonal distinction with respect to an appropriate scalar variable. This aspect of the project is discussed in detail in appendix III which contains also the same preliminary results.

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List of publications

[1] S. Byggstoyl, W. Kollmann: "Stress transport in the rotational and irrotational zones of turbulent shear flows," submitted for publication . (Appendix I).

[2] S. Byggstoyl, W. Kollmann, "A closure model for conditional stress equations and its application to turbulent shear flows," submitted for publication. (Appenidx II).

Professional personnel

1. S. Byggstoyl, Post Doctoral Fellow from Trondheim, Norway. Ph.D.--thesis: "Mathematical Modelling of Turbulent Structure Effects in Reacting and Nonreacting Flows," August 1984, NTH--Trandheim.

2. M. Mortazavi, Ph.D. student, graduated from the Department of Chemical Engineering, UC Davis, 1984.

3. W. Kollmann, Professor, Department of Mechanical Engineering, UC Davis, Principal Investigator.

Appendix I: Stress transport in the rotational and irrotational zones of turbulent shear flows

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Abstract: The transport equations for intermittency factor and conditioned moments are analyzed for turbulent shear flows with free boundaries. Conditions are established for molecular diffusion to dominate the progression of the turbulent zone. The terms related to the dynamics of the interface for mean velocity and apparent stresses in turbulent and nonturbulent zones are shown to be linked by a local relation and the limit of large distance from the turbulent region for the nonturbulent zone stress equation is given.

Stress transport in the rotational and irrotational zones of turbulent shear flows

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1. Introduction

Turbulent shear flows with free boundaries such as jets and mixing layers show rotational and irrotational fluctuations of velocity near the free boundary. This was established by Corrsin and Kistler [1], [2] expermentally for several boundary-layer-type flows. Since then a large number of experimental results became available (see references given in [3]). The theoretical investigation of intermittently turbulent flows started with Corrsin and Kistler [2], Phillips [4], [7] Stewart [5], Corrsin and Phillips [6], dealing in particular with interface properties and irrotational fluctuations. Lumley [8], [9] introduced the statistics of multi-valued random functions from the treatment of interface fluctuations and developed closure ideas. The first closure model based on first and second order moments was published by Libby [10], [11]. Dopazo [12] and Dopazo and O'Brien [13] established the theoretical representation of interface related processes in conditioned moment equations. Pope [14] showed, that conditioning can be easily applied to pdf-equations.

The present paper extends the analysis of ref. [3] to stress transport in the nonturbulent zone of shear flows with free boundaries. The condition of irrotationality in terms of the Corrsin-Kistler equation is

exploited to establish limit forms for production and diffusion of stress as the intermittency factor approaches zero. The results obtained will be used in a companion paper to construct a second order closure model based on [15],[16],[17], which includes the transport equations for the apparent stresses in the nonturbulent zone.

2. Conditional statistics and intermittency factor.

The statistical description of turbulent flows can be refined by conditioning of expectations to capture particular properties of the flow in an explicit fashion [3],[12]. For this purpose a non-negative scalar variable $\phi(\underline{x},t)$ is selected in such a way, that a local condition being satisfied at a point $(\underline{x},t)$ of the flow field corresponds to

$$\phi(\underline{x},t) \geq h > 0$$

where h is the threshold value, and

$$\phi(\underline{x},t) < h$$

corresponds to the condition being violated. In the present case the condition is that the flow is turbulent at the point considered and thus [3]

$$\phi(\underline{x},t) = \omega_k'\omega_k' \geq 0$$

is taken as discriminating variable. It should be noted that many other conditions can be considered (such as hot-cold, burnt-unburnt, colored-clear) which would lead to different variables $\phi(\underline{x},t)$.

Conditioning of flow variables can be done with the aid of the indicator function $I(\underline{x},t)$

$$I(\underline{x},t) = \begin{cases} 1 & \text{for } \phi(\underline{x},t) \geq h \\ 0 & \text{otherwise} \end{cases}$$

The locations $\underline{x}$ where

$$S(\underline{x},t) = \phi(\underline{x},t) - h = 0 \tag{1}$$

form a surface separating turbulent from non-turbulent zones. This surface propagates with velocity v_α^s and progresses relative to the fluid in its normal direction n_α (positive into turbulent zone) with speed V [12]

$$n_\alpha V = v_\alpha - v_\alpha^s \tag{2}$$

Then follows [3]

$$\frac{\partial I}{\partial t} = - v_\alpha^s n_\alpha \delta^*(S) \tag{3}$$

and

$$\frac{\partial I}{\partial x_\alpha} = n_\alpha \delta^*(S) \tag{4}$$

where

$$\delta^*(S) = |\nabla S| \delta(S) \tag{5}$$

Conditional moments can now be defined for the turbulent zone

$$\tilde{\bar{\varphi}} = \frac{\langle I\varphi\rangle}{\gamma} \quad , \quad \varphi^{*} = \varphi - \tilde{\bar{\varphi}} \tag{6}$$

and the non-turbulent zone

$$\tilde{\tilde{\varphi}} \equiv \frac{\langle (1-I)\varphi\rangle}{1-\gamma} \quad , \quad \varphi^{o} = \varphi - \tilde{\tilde{\varphi}} \tag{7}$$

where the intermittency factor is given by

$$\gamma = \langle I\rangle \tag{8}$$

Points on the interface move with velocity v_α^s thus

$$\frac{\partial I}{\partial t} + v_\alpha^s \frac{\partial I}{\partial x_\alpha} = 0 \tag{9}$$

which can be recast in terms of fluid velocity v_α and the relative progression velocity of the interface.

$$\frac{\partial I}{\partial t} + v_\alpha \frac{\partial I}{\partial x_\alpha} = n_\alpha V \frac{\partial I}{\partial x_\alpha} \tag{10}$$

Averaging leads to the exact equation for the intermittency factor [3]

$$\frac{\partial \gamma}{\partial t} + \frac{\partial}{\partial x_\alpha}(\tilde{\bar{v}}_\alpha \gamma) = \langle V\delta^{*}(s)\rangle \tag{11}$$

Introducing the unconditional mean as convection velocity yields

$$\frac{\partial \gamma}{\partial t} + \frac{\partial}{\partial x_\alpha}(\bar{v}_\alpha \gamma) = -\frac{\partial}{\partial x_\alpha}\left[\gamma(1-\gamma)(\tilde{\bar{v}}_\alpha - \tilde{\tilde{v}}_\alpha)\right] + \langle V\delta^{*}(s)\rangle \tag{12}$$

This form of the transport equation for the intermittency factor shows, that the turbulent diffusion of γ is due to the relative movement of

turbulent and non-turbulent zones if a point moving with the unconditional mean velocity is followed. The intermittency source S_γ

$$S_\gamma \equiv \langle V \delta(S) \rangle \tag{13}$$

is the rate at which the volume of the zone $\phi \geq h$ grows per unit volume of fluid. A detailed analysis [3] shows that S_γ can be represented in several forms. The dependence of this growth rate S_γ on the scalar variable discriminating between the zones can be shown explicitly [3]. If ϕ satisfies

$$\frac{\partial \phi}{\partial t} + u_\alpha \frac{\partial \phi}{\partial x_\alpha} = \frac{\partial}{\partial x_\alpha}\left(\Gamma \frac{\partial \phi}{\partial x_\alpha}\right) + Q \tag{14}$$

then follows for S_γ

$$S_\gamma = \left\langle \frac{DS}{Dt} \delta(S) \right\rangle$$

where D/Dt denotes the substantial derivative. The transport equation for ϕ yields then the representation

$$S_\gamma = \left\langle \left[\frac{\partial}{\partial x_\alpha}\left(\Gamma \frac{\partial \phi}{\partial x_\alpha}\right) + Q\right] \delta(S) \right\rangle \tag{15}$$

This result shows that two mechanisms for growth are present namely diffusion and sources. The influence of the source Q on S_γ depends crucially on the threshold level h and the limit of Q as ϕ approaches zero. If

$$\lim_{h \to 0} \langle Q \delta(S) \rangle \cong O(h^\alpha)$$

with an exponent $O(\alpha) \geq 1$, then is the effect of $\langle Q \delta(S) \rangle$ on S_γ negligible if h is sufficiently small. For this case can

be concluded, that discrimination with a conserved scalar yields the same result as discrimination with a non-conserved scalar. This does not hold, if the threshold h is raised to a level with large rates of production or destruction of ϕ. An estimate for the threshold level h, such that the diffusive part of S_γ dominates, can be obtained, if the source term Q can be represented as power in ϕ

$$S(\phi) \cong A\phi^n$$

for small ϕ. The source term S_γ is

$$S_\gamma = S_\gamma^D + S_\gamma^Q$$

where S_γ^D and S_γ^D can be written in terms of conditional expectations

$$S_\gamma^D \equiv \langle \frac{\partial}{\partial x_\alpha}(\Gamma \frac{\partial \phi}{\partial x_\alpha}) / \phi = h \rangle P(h) \tag{16}$$

and

$$S_\gamma^Q \equiv \langle Q / \phi = h \rangle P(h) \tag{17}$$

if Q is a power of ϕ we obtain

$$S_\gamma^Q = A h^n P(h)$$

The diffusive part S_γ^D can be estimated by

$$O(S_\gamma^D) = P(h) \Gamma \frac{h}{\ell^2}$$

The length scale ℓ can be related to the micro-scale L of the flow, if it is required, that the dominant production and destruction terms in

the equation for scalar dissipation have the same order of magnitude. Then follows

$$O\left(\frac{\ell}{L}\right) = \left(\frac{\Gamma}{\nu}\right)^{\frac{1}{4}} Re^{-\frac{5}{8}}$$

and the relative order of magnitude of S_γ^Q to S_γ^D is

$$O\left(\frac{S_\gamma^Q}{S_\gamma^D}\right) = A\, h^{n-1} \frac{L}{U} \left(\frac{\nu}{\Gamma}\right)^{\frac{1}{2}} Re^{-\frac{1}{4}} \tag{18}$$

where L and U are the macro-scales for length and velocity and $Re = UL/\nu$. If this ratio is required to be less than unity, we get

$$h^{n-1} < Re^{\frac{1}{4}} \left(\frac{\Gamma}{\nu}\right)^{\frac{1}{2}} \frac{U}{LA}$$

Thus the following conclusions are reached. If $Q(\phi)$ approaches zero faster than ϕ, the diffusive part of S_γ is always dominant for sufficiently large Re-number. If n is less than unity however, we find

$$h^{1-n} > Re^{-\frac{1}{4}} \left(\frac{\nu}{\Gamma}\right)^{\frac{1}{2}} \frac{LA}{U}$$

and the threshold level h for ϕ is restricted by the Re-number from below. If h violates this inequality the source term S_γ^Q will dominate the development of the intermittency factor.

The structure of the diffusive contribution of S_γ can be illustrated for the case of Gaussian statistics of the scalar ϕ. This example can be viewed as ϕ being lognormal or $\ln \phi$ being an unbounded discriminator. The mean $\langle\phi\rangle$ and the variance $\langle\phi'^2\rangle$ are taken as constants and the spatial correlation coefficient

$$\rho(r_\alpha) = 1 + \frac{1}{2} r_\alpha r_\beta \frac{\partial^2 \rho}{\partial r_\alpha \partial r_\beta}(0) + h.o.t.$$

is developed in a Taylor series. Writing for the source term

$$\left\langle \frac{\partial}{\partial x_\alpha}\left(\Gamma \frac{\partial \phi}{\partial x_\alpha}\right)\delta(\phi - h)\right\rangle = \lim_{\Delta x_\alpha \to 0} \frac{1}{\Delta x_\alpha^2}\left\{\left\langle \Gamma \phi(x_\alpha + \Delta x_\alpha)\delta(\phi - h)\right\rangle\right.$$

$$\left. + \left\langle \Gamma \phi(x_\alpha - \Delta x_\alpha)\delta(\phi - h)\right\rangle - 2\left\langle \Gamma \phi\, \delta(\phi - h)\right\rangle\right\}$$

and introducing the assumption that $\phi(x_\alpha \pm \Delta x_\alpha)$ and $\phi(x_\alpha)$ are Gaussian-distributed, we get after some algebra

$$\left\langle \frac{\partial}{\partial x_\alpha}\left(\Gamma \frac{\partial \phi}{\partial x_\alpha}\right)\delta(\phi - h)\right\rangle = -P_\phi(h)\left(\langle\phi\rangle - h\right)\Gamma \frac{\partial^2 \rho}{\partial r_\alpha \partial r_\alpha}(0) \qquad (19)$$

This result has several interesting properties. First it shows that the intermittency source depends on the probability for $\phi(\underline{x}) = h$ at the point $\underline{x}$ and therefore on the threshold value h for the discriminating scalar ϕ . Furthermore it becomes evident from (19), that for the limit of infinite Re/Sc-numbers the intermittency source will approach a nonzero and bounded constant, because

$$\Gamma \frac{\partial^2 \rho}{\partial r_\alpha \partial r_\alpha}(0) = -\langle \phi'^2\rangle^{-1} \varepsilon_\phi$$

becomes independent of Re/Sc-numbers by virtue of the same arguments that apply to viscous dissipation. Hence can (19) be recast as follows for the example

$$S_\gamma^D = \left\langle \frac{\partial}{\partial x_\alpha}\left(\Gamma \frac{\partial \phi}{\partial x_\alpha}\right)\delta(\phi - h)\right\rangle = P_\phi(h)\, \frac{\left(\langle\phi\rangle - h\right)}{\langle \phi'^2\rangle}\, \varepsilon_\phi \qquad (20)$$

showing explicitly the dependence on the scalar time scale. The influence on the velocity fluctuations enters via the pdf P_ϕ of the scalar ϕ ,

which is transported and diffused by the velocity field. Finally, it should be noted that the intermittency source is not positive definite, but can become negative for $\langle\phi\rangle < h$.

3. Mean velocities in turbulent and non-turbulent zones.

The balance of momentum can be averaged conditionally thus leading to the equations for zone-averaged mean velocities. The turbulent zone mean velocity $\bar{\bar{v}}_\alpha$ satisfies

$$\frac{\partial}{\partial t}\bar{\bar{v}}_\alpha + \bar{v}_\beta \frac{\partial \bar{\bar{v}}_\alpha}{\partial x_\beta} = -\frac{1}{\gamma}\frac{\partial}{\partial x_\beta}\left(\gamma\, \overline{v^*_\alpha v^*_\beta}\right) + \nu \frac{\partial^2 \bar{\bar{v}}_\alpha}{\partial x_\beta \partial x_\beta}$$

$$-\frac{1}{\rho}\frac{\partial \bar{\bar{p}}}{\partial x_\alpha} + (1-\gamma)(\tilde{v}_\beta - \bar{\bar{v}}_\beta)\frac{\partial \bar{\bar{v}}_\alpha}{\partial x_\beta} + \frac{1}{\gamma} S^*_\alpha \qquad (21)$$

and for the non-turbulent zone follows

$$\frac{\partial}{\partial t}\tilde{v}_\alpha + \bar{v}_\beta \frac{\partial \tilde{v}_\alpha}{\partial x_\beta} = -\frac{1}{1-\gamma}\frac{\partial}{\partial x_\beta}\left[(1-\gamma)\,\widetilde{v^\circ_\alpha v^\circ_\beta}\right] + \nu \frac{\partial^2 \tilde{v}_\alpha}{\partial x_\beta \partial x_\beta}$$

$$-\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_\alpha} - \gamma(\tilde{v}_\beta - \bar{\bar{v}}_\beta)\frac{\partial \tilde{v}_\alpha}{\partial x_\beta} - \frac{1}{1-\gamma} S^\circ_\alpha \qquad (22)$$

Both equations contain a term representing the effect of the interface movement on the average momentum in the two zones. These interface terms can be given in the form

$$S^i_\alpha = \frac{1}{\rho}\langle p' n_\alpha \delta^*(S)\rangle + F^i_\alpha \qquad (23)$$

and

$$F^i_\alpha = \langle v'_\alpha V \delta^*(S)\rangle - \nu\frac{\partial}{\partial x_\beta}\langle v'_\alpha n_\beta \delta^*(S)\rangle - \nu\langle n_\beta \frac{\partial v'_\alpha}{\partial x_\beta}\delta^*(S)\rangle \qquad (24)$$

where $i = *$ for the turbulent zone quantities and $i = 0$ for the non-turbulent zone. The interface terms S_α^*, S_α^0 and F_α^*, F_α^0 are not independent but linked due to

$$\bar{v}_\alpha = \gamma \tilde{\bar{v}}_\alpha + (1-\gamma)\tilde{\bar{v}}_\alpha$$

Combination of (21) with (22) must reproduce the equation for the unconditioned mean velocity. Thus follows [16] after same manipulations

$$F_\alpha^* - F_\alpha^0 = (\tilde{\bar{v}}_\alpha - \bar{\bar{v}}_\alpha)(S_\gamma - \nu\Delta\gamma) - 2\nu \frac{\partial\gamma}{\partial x_\beta}\frac{\partial}{\partial x_\beta}(\tilde{\bar{v}}_\alpha - \bar{\bar{v}}_\alpha) \qquad (25)$$

and

$$\langle p^* n_\alpha \delta^*(s)\rangle - \langle p^0 n_\alpha \delta^*(s)\rangle = (\tilde{\bar{p}} - \bar{\bar{p}})\frac{\partial\gamma}{\partial x_\alpha} \qquad (26)$$

Equations (25) and (26) must be satisfied for closure expressions relating F_α^i and $\langle p^i n_\alpha \delta^*(s)\rangle$ to other quantities and are called consistency conditions.

So far no use has been made of the fact that the non-turbulent zone is irrotational. Following Corrsin and Kistler [2] it is required

$$|\omega_\alpha| = O(h^{\frac{1}{2}}) \quad \text{for } I = 0$$

or

$$(1-I)\left(\frac{\partial v_\alpha}{\partial x_\beta} - \frac{\partial v_\beta}{\partial x_\alpha}\right) = O(h^{\frac{1}{2}}) \qquad (27)$$

where h is the threshold value for the turbulent-nonturbulent discrimination. This relation can be applied to establish the properties of transport equations for moments in the non-turbulent zone as the outer

edge of the flow is approached (corresponding to γ approaching zero). Averaging (27) leads to the Corrsin-Kistler equation

$$(1-\gamma)\left(\frac{\partial\tilde{\bar{v}}_\alpha}{\partial x_\beta} - \frac{\partial\tilde{\bar{v}}_\beta}{\partial x_\alpha}\right) + \langle(v_\alpha^\circ n_\beta - v_\beta^\circ n_\alpha)\delta^*(S)\rangle = O(h^{\frac{1}{2}}) \qquad (28)$$

of first order. The interface term in (28) can readily be shown to satisfy the following conditions:

$$\lim_{\gamma\to 0}\langle(v_\alpha^\circ n_\beta - v_\beta^\circ n_\alpha)\delta^*(S)\rangle = 0$$

and

$$\lim_{\gamma\to 1}\langle(v_\alpha^\circ n_\beta - v_\beta^\circ n_\alpha)\delta^*(S)\rangle = 0$$

The first limit follows from the fact that in the non-turbulent flow field the mean vorticity approaches zero as the distance from the turbulent zone grows. The second limit follows directly from (28). Therefore is a representation of the interface term in (28) in the form

$$\langle(v_\alpha^\circ n_\beta - v_\beta^\circ n_\alpha)\delta^*(S)\rangle = \gamma(1-\gamma)K_{\alpha\beta}(\underline{x},t) \qquad (29)$$

possible with an unknown bounded function $K_{\alpha\beta}$. These properties of the Corrsin-Kistler equation will be used in the discussion of the dynamics of the apparent stress tensors in turbulent and non-turbulent zones.

Corrsin-Kistler equations of higher order can be obtained from (27) by multiplication with fluctuating components and averaging. Thus follows

$$\frac{\partial}{\partial x_\beta}\left[(1-\gamma)\widetilde{\overline{v_\alpha^\circ v_\beta^\circ}}\right] - \frac{\partial}{\partial x_\alpha}\left[(1-\gamma)\tilde{\bar{k}}^\circ\right] = \langle(k^\circ n_\alpha - v_\alpha^\circ v_\beta^\circ n_\beta)\delta^*(S)\rangle + O(h) \qquad (30)$$

where $k^\circ = \frac{1}{2}v_\alpha^\circ v_\alpha^\circ$ denotes the kinetic energy in the

non-turbulent zone. For the limit $\gamma \to 0$ the relation [2],[5]

$$\frac{\partial}{\partial x_\beta} \widetilde{v_\alpha^o v_\beta^o} = \frac{\partial \tilde{\tilde{k}}^o}{\partial x_\alpha}$$

is obtained, which shows that the effect of the apparent stress on the mean velocity in the non-turbulent zone becomes analogous to the pressure-gradient.

4. Apparent stresses in turbulent and non-turbulent zones.

The momentum balances and mass conservation can be multiplied with appropriate fluctuations and the indicator function and averaged to establish the transport equations for the zonal stress tensors. Thus the equation for the stress tensor in the turbulent zone

$$\overline{\overline{v_\alpha^* v_\beta^*}} = \frac{\langle I v_\alpha^* v_\beta^* \rangle}{\gamma} \tag{31}$$

is obtained in the form

$$\frac{\bar{\bar{D}}}{Dt} \overline{\overline{v_\alpha^* v_\beta^*}} = - \overline{\overline{v_\alpha^* v_\gamma^*}} \frac{\partial \bar{\bar{v}}_\beta}{\partial x_\gamma} - \overline{\overline{v_\beta^* v_\gamma^*}} \frac{\partial \bar{\bar{v}}_\alpha}{\partial x_\gamma} - \bar{\bar{\varepsilon}}_{\alpha\beta}$$

$$+ \frac{1}{\gamma} \frac{\partial}{\partial x_\gamma} \left[\gamma \, \mathcal{D}^*_{\alpha\beta\gamma} + \nu \frac{\partial}{\partial x_\gamma} (\gamma \overline{\overline{v_\alpha^* v_\beta^*}}) \right] + \frac{1}{\rho} \overline{\overline{p^* \left(\frac{\partial v_\alpha^*}{\partial x_\beta} + \frac{\partial v_\beta^*}{\partial x_\alpha} \right)}}$$

$$- \frac{1}{\gamma} S_\gamma \overline{\overline{v_\alpha^* v_\beta^*}} + \frac{1}{\gamma} S^*_{\alpha\beta} \tag{32}$$

The tensor of apparent stresses in the non-turbulent zone

$$\widetilde{v_\alpha^o v_\beta^o} = \frac{\langle (1 - I) v_\alpha^o v_\beta^o \rangle}{1 - \gamma} \tag{33}$$

is governed by

$$\frac{\tilde{\tilde{D}}}{Dt}\,\widetilde{\overline{u^\circ_\alpha u^\circ_\beta}} = -\widetilde{\overline{u^\circ_\alpha u^\circ_\gamma}}\,\frac{\partial \tilde{\tilde{U}}_\beta}{\partial x_\gamma} - \widetilde{\overline{u^\circ_\beta u^\circ_\gamma}}\,\frac{\partial \tilde{\tilde{U}}_\alpha}{\partial x_\gamma} - \tilde{\tilde{\varepsilon}}_{\alpha\beta}$$

$$+ \frac{1}{1-\gamma}\frac{\partial}{\partial x_\gamma}\left[(1-\gamma)D^\circ_{\alpha\beta\gamma} + \nu\frac{\partial}{\partial x_\gamma}\left((1-\gamma)\widetilde{\overline{u^\circ_\alpha u^\circ_\beta}}\right)\right] +$$

$$+ \frac{1}{\rho}\widetilde{\overline{p^\circ\left(\frac{\partial u^\circ_\alpha}{\partial x_\beta} + \frac{\partial u^\circ_\beta}{\partial x_\alpha}\right)}} + \frac{1}{1-\gamma}S_\gamma\,\widetilde{\overline{u^\circ_\alpha u^\circ_\beta}} - \frac{1}{1-\gamma}S^\circ_{\alpha\beta} \qquad (34)$$

The substantial derivative in (32) and (34) is defined by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + U_\alpha\frac{\partial}{\partial x_\alpha}$$

where the superscript applies to the velocity.

Both stress equations (32) and (34) contain dissipation rates

$$\tilde{\varepsilon}_{\alpha\beta} = 2\nu\,\widetilde{\overline{\frac{\partial u^*_\alpha}{\partial x_\gamma}\frac{\partial u^*_\beta}{\partial x_\gamma}}} \qquad (35)$$

and

$$\tilde{\tilde{\varepsilon}}_{\alpha\beta} = 2\nu\,\widetilde{\overline{\frac{\partial u^\circ_\alpha}{\partial x_\gamma}\frac{\partial u^\circ_\beta}{\partial x_\gamma}}} \qquad (36)$$

Their importance depends on the discriminating scalar. For the present case of distinction between turbulent and non-turbulent zones becomes the dissipation $\tilde{\tilde{\varepsilon}}_{\alpha\beta}$ in the non-turbulent zone negligible and the unconditioned rate of dissipation is related to the conditioned rates by

$$\varepsilon_{\alpha\beta} \cong \gamma\,\tilde{\varepsilon}_{\alpha\beta}$$

If the discriminating scalar is chosen to afford distinction between other properties of the flow such as hot and cold, then both zones can be turbulent and both dissipation rates $\bar{\bar{\varepsilon}}_{\alpha\beta}$ and $\tilde{\tilde{\varepsilon}}_{\alpha\beta}$ are important.

The turbulent fluxes in (32) and (34 are defined by

$$\mathcal{D}^*_{\alpha\beta\delta} \equiv -\overline{\overline{v^*_\alpha v^*_\beta v^*_\delta}} - \delta_{\alpha\delta}\overline{\overline{v^*_\beta p^*}} - \delta_{\beta\delta}\overline{\overline{v^*_\alpha p^*}} \qquad (37)$$

and

$$\mathcal{D}^\circ_{\alpha\beta\delta} \equiv -\widetilde{\overline{v^\circ_\alpha v^\circ_\beta v^\circ_\delta}} - \delta_{\alpha\delta}\widetilde{\overline{v^\circ_\beta p^\circ}} - \delta_{\beta\delta}\widetilde{\overline{v^\circ_\alpha p^\circ}} \qquad (38)$$

The existence of a fluctuating interface separating turbulent and non-turbulent zones leads to the transport of apparent stress through the interface, transport due to interface movement, production of apparent stress due to interface fluctuations. These effects are represented by the term group $S^i_{\alpha\beta}$ defined by

$$S^i_{\alpha\beta} \equiv \frac{1}{\rho}\langle p^i(v^i_\alpha n_\beta + v^i_\beta n_\alpha)\delta^*(s)\rangle + F^i_{\alpha\beta} \quad , i = *, 0 \qquad (39)$$

and

$$F^i_{\alpha\beta} \equiv \langle v^i_\alpha v^i_\beta V\delta^*(s)\rangle - \nu\langle n_\delta \frac{\partial}{\partial x_\delta}(v^i_\alpha v^i_\beta)\delta^*(s)\rangle$$

$$- \nu\frac{\partial}{\partial x_\delta}\langle v^i_\alpha v^i_\beta n_\delta \delta^*(s)\rangle \qquad , i = *, 0 \qquad (40)$$

The interface terms $S^*_{\alpha\beta}$ and $S^\circ_{\alpha\beta}$ are not independent. The relation between conditioned and unconditioned stress tensors

$$\overline{v'_\alpha v'_\beta} = \gamma\overline{\overline{v^*_\alpha v^*_\beta}} + (1-\gamma)\widetilde{\overline{v^\circ_\alpha v^\circ_\beta}} + \gamma(1-\gamma)(\tilde{\bar{v}}_\alpha - \bar{\bar{v}}_\alpha)(\tilde{\bar{v}}_\beta - \bar{\bar{v}}_\beta) \qquad (41)$$

leads to a consistency condition for the $S'_{\alpha\beta}$. Introducing the abbreviations

$$\Delta v_\alpha \equiv \tilde{\tilde{v}}_\alpha - \bar{\tilde{v}}_\alpha$$

and

$$\Delta p = \tilde{\tilde{p}} - \bar{\tilde{p}}$$

we obtain after some manipulations using (32), (34), (41) the following relation

$$\begin{aligned}
S^*_{\alpha\beta} - S^\circ_{\alpha\beta} &= (1-2\gamma)\,\Delta v_\alpha \Delta v_\beta S_\gamma - \gamma(\Delta v_\alpha S^\circ_\beta + \Delta v_\beta S^\circ_\alpha) \\
&- (1-\gamma)(\Delta v_\alpha S^*_\beta + \Delta v_\beta S^*_\alpha) + (1-\gamma)\,\widetilde{v^\circ_\alpha v^\circ_\beta}\,\frac{\partial}{\partial x_\gamma}(\gamma \Delta v_\gamma) \\
&- \gamma\,\overline{\widetilde{v^*_\alpha v^*_\beta}}\,\frac{\partial}{\partial x_\gamma}[(1-\gamma)\Delta v_\gamma] + \Delta p\Big\{(1-2\gamma)\Big(\Delta v_\alpha \frac{\partial \gamma}{\partial x_\beta} + \Delta v_\beta \frac{\partial \gamma}{\partial x_\alpha}\Big) \\
&- \langle n_\alpha(\gamma v^\circ_\beta + (1-\gamma)v^*_\beta)\,\delta(S)\rangle - \langle n_\beta(\gamma v^\circ_\alpha + (1-\gamma)v^*_\alpha)\,\delta(S)\rangle\Big\} \\
&- 2\nu\gamma(1-\gamma)\frac{\partial}{\partial x_\gamma}\Delta v_\alpha \frac{\partial}{\partial x_\gamma}\Delta v_\beta - 2\nu(1-2\gamma)\frac{\partial \gamma}{\partial x_\gamma}\frac{\partial}{\partial x_\gamma}(\Delta v_\alpha \Delta v_\beta) - \nu \Delta v_\alpha \Delta v_\beta \frac{\partial^2}{\partial x_\gamma \partial x_\gamma}[\gamma(1-\gamma)]
\end{aligned} \tag{42}$$

This rather complicated relation can be somewhat simplified if the limit case of high Re-numbers is considered. Then follows that the viscous terms in (42) can be neglected, because they scale with the mean fields.

The transport of apparent stress in the zone, where the discriminating scalar ϕ is below the threshold level, merits discussion in particular if distinction between turbulent and non-turbulent zones is considered. We consider the terms on the right hand side of (34) for this case in turn. The production of stress due to the interaction of stresses and mean deformation rates

$$\tilde{\tilde{P}}_{\alpha\beta} \equiv - \widetilde{\overline{v_\alpha^\circ v_\gamma^\circ}} \frac{\partial \tilde{\tilde{v}}_\beta}{\partial x_\gamma} - \widetilde{\overline{v_\beta^\circ v_\gamma^\circ}} \frac{\partial \tilde{\tilde{v}}_\alpha}{\partial x_\gamma}$$

can be analyzed using the Corrsin-Kistler relation (28). Taking (29) into account we find

$$\tilde{\tilde{P}}_{\alpha\beta} = - \widetilde{\overline{v_\alpha^\circ v_\sigma^\circ}} \frac{\partial \tilde{\tilde{v}}_\sigma}{\partial x_\beta} - \widetilde{\overline{v_\beta^\circ v_\sigma^\circ}} \frac{\partial \tilde{\tilde{v}}_\sigma}{\partial x_\alpha} + \gamma \left(\widetilde{\overline{v_\alpha^\circ v_\sigma^\circ}} K_{\beta\sigma} + \widetilde{\overline{v_\beta^\circ v_\sigma^\circ}} K_{\alpha\sigma} \right)$$

Hence we find for the limit $\gamma \to 0$

$$\tilde{\tilde{P}}_{\alpha\beta} \to - \widetilde{\overline{v_\alpha^\circ v_\sigma^\circ}} \frac{\partial \tilde{\tilde{v}}_\sigma}{\partial x_\beta} - \widetilde{\overline{v_\beta^\circ v_\sigma^\circ}} \frac{\partial \tilde{\tilde{v}}_\sigma}{\partial x_\alpha}$$

On the other hand is for the limit $\gamma \to 1$ the original form of $\tilde{\tilde{P}}_{\alpha\beta}$ relevant because the non-turbulent patches become increasingly rare in this limit and will follow the motion of the surrounding turbulent fluid. It is instructive to write out the production terms for both limits in case of a plane parabolic flow (i.e., plane jet).

Limit: $\gamma \to 1$	Limit: $\gamma \to 0$
$\tilde{\tilde{P}}_{11} = -2\, \widetilde{\overline{v_1^\circ v_2^\circ}} \dfrac{\partial \tilde{\tilde{v}}_1}{\partial x_2}$	$\tilde{\tilde{P}}_{11} = 0$
$\tilde{\tilde{P}}_{22} = 0$	$\tilde{\tilde{P}}_{22} = -2\, \widetilde{\overline{v_1^\circ v_2^\circ}} \dfrac{\partial \tilde{\tilde{v}}_1}{\partial x_2}$
$\tilde{\tilde{P}}_{33} = 0$	$\tilde{\tilde{P}}_{33} = 0$
$\tilde{\tilde{P}}_{12} = - \widetilde{\overline{v_2^{\circ 2}}} \dfrac{\partial \tilde{\tilde{v}}_1}{\partial x_2}$	$\tilde{\tilde{P}}_{12} = - \widetilde{\overline{v_1^{\circ 2}}} \dfrac{\partial \tilde{\tilde{v}}_1}{\partial x_2}$

Note that for parabolic flows derivatives with respect to x_1 , can be neglected (boundary layer assumption). It becomes apparent from the table above, that energy is fed into the stress component $\widetilde{\overline{v_2^{o2}}}$ in the limit $\gamma \to 0$, whereas for $\gamma \to 1$ $\widetilde{\overline{v_1^{o2}}}$ receives the energy as in the turbulent zone. The limit $\gamma \to 0$ is consistent with the relation among the normal stresses (valid for $\gamma \to 0$)

$$\widetilde{\overline{v_2^{o2}}} = \widetilde{\overline{v_1^{o2}}} + \widetilde{\overline{v_3^{o2}}} \tag{43}$$

in the nonturbulent zone which was obtained by Phillip [4] and Stewart [5].

The diffusive flux $\mathcal{D}^o_{\alpha\beta\gamma}$ contains the triple correlations of velocity, which can be analyzed by means of the Corrsin-Kistler equation for triple moments

$$\frac{\partial}{\partial x_\gamma}\left[(1-\gamma)\widetilde{\overline{v_\alpha^o v_\beta^o v_\gamma^o}}\right] - \frac{\partial}{\partial x_\alpha}\left[(1-\gamma)\widetilde{\overline{v_\beta^o k^o}}\right] - \frac{\partial}{\partial x_\beta}\left[(1-\gamma)\widetilde{\overline{v_\alpha^o k^o}}\right]$$

$$+ 2(1-\gamma)\widetilde{\overline{k^o s^o_{\alpha\beta}}} = -\left\langle (v_\alpha^o v_\beta^o v_\gamma^o n_\gamma - v_\alpha^o k^o n_\beta - v_\beta^o k^o n_\alpha)\delta^*(s)\right\rangle$$

$$+ O(h^{3/2}) \tag{44}$$

where $S_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v_\alpha}{\partial x_\beta} + \frac{\partial v_\beta}{\partial x_\alpha}\right)$ denotes the rate of strain.

For the limit $\gamma \to 0$ the relation

$$\frac{\partial}{\partial x_\gamma}\widetilde{\overline{v_\alpha^o v_\beta^o v_\gamma^o}} = \frac{\partial}{\partial x_\alpha}\widetilde{\overline{v_\beta^o k^o}} + \frac{\partial}{\partial x_\gamma}\widetilde{\overline{v_\alpha^o k^o}} - 2\widetilde{\overline{k^o s^o_{\alpha\beta}}}$$

is obtained. Hence is the divergence of the flux of apparent stress for this limit composed of gradients of the flux of kinetic energy and the correlation of kinetic energy with the fluctuating strain rate. More

distance from the interface increases. Phillips [7] showed that for large y

$$O(v_\alpha^\circ) = y^{-2}$$

where y is the normal distance from the interface. Using the Bernoulli equation, which is valid in the non-turbulent zone only, it follows that

$$O(p^\circ) = y^{-4}$$

These estimates can be used to assess the relative importance of several terms in the moment equations for the non-turbulent zone for large distance from the turbulent zone or as $\gamma \rightarrow 0$. It is worth noting that irrotational fluctuations are induced by boundaries. Thus is statistical homogeneity not possible and decay estimates have to be used instead.

The interface terms in (34) show that the growth of the turbulent zone leads to an increase of non-turbulent zone stresses by

$$\frac{1}{1-\gamma} S_\gamma \overline{\widetilde{v_\alpha^\circ v_\beta^\circ}}$$

which may be cancelled by the complex production/destruction group of terms $S_{\alpha\beta}^\circ$, as the consistency relation (42) indicates, which contains the intermittency and conditioned momentum sources.

5. Conclusions.

The transport equations for intermittency factor and conditioned moments were set up and their properties were analyzed. The conclusions

can be summarized as follows.

(1) Intermittency factor: The intermittency source was shown to be composed of two different terms representing growth due to molecular diffusion and production of the scalar variable used for discrimination between the zones. The diffusive term is the dominant mechanism for growth if the source term approaches zero faster than the first power of the scalar. Further was shown that the intermittency source is not always positive.

(2) Conditioned mean velocity: The equations for turbulent and non-turbulent zone mean velocity contain source terms describing momentum transport through the interface and production/destruction of mean momentum due to interface movement. These source terms are not independent but linked by a local consistency relation, which shows that the difference between these sources is proportional to intermittency source and the difference of conditioned mean velocities. The Corrsin-Kistler equation introduces the condition of irrotationality in the non-turbulent zone and proves that for the limit $\gamma \to 0$ the effect of apparent stress on the non-turbulent mean velocity becomes analogous to the mean pressure-gradient.

(3) Conditioned stress tensors: The transport equations for the apparent stress tensors in turbulent and non-turbulent zones contain source terms describing the production/destruction of stress due to the interface fluctuations. These source terms are not independent but linked by a local consistency relation as in case of the mean velocities. The analysis of the stress transport equation in the

non-turbulent zone shows that production in the limit $\gamma \rightarrow 0$ for boundary-layer-type conditions shifts from $\widetilde{\overline{v_1'^2}}$ to $\widetilde{\overline{v_2'^2}}$, thus confirming the dominance of component $\widetilde{\overline{v_2'^2}}$ for this limit. The Corrsin-Kistler relation for triple correlations shows, that under the same conditions becomes diffusion of $\widetilde{\overline{v_2'^2}}$ dominant, if the correlation of kinetic energy and strain rate is weak.

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Appendix II: A closure model for conditioned stress equations and its application to turbulent shear flows

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Abstract: A second order closure model based on intermittency factor and conditioned moments is developed. The transport equations for the nonturbulent zone stresses are included in the model. The resulting model is then compared with measurements in several shear flows and satisfactory agreement between calculation and experiment is obtained.

A closure model for conditioned stress equations

and its application to turbulent shear flows

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(1) Introduction

Turbulent shear flows with free boundaries such as jets, wakes, mixing layers show an intermittent character in the fluctuations of velocity changing from rotational to irrotational and vice versa [1]. The prediction of this type of shear flows can be based on unconditional moments [2],[3] on conditional moments [4],[5],[6]. The latter case is considered here, which allows calculation of the intermittency factor and statistical moments characterizing the fluctuations in the individual zones. The closure model developed in this paper is based on the work presented in [7],[8],[9]. The new part is the inclusion of transport equations for the apparent stresses in the nonturbulent zone. The properties of turbulent and nonturbulent zone moments discussed in [9] are used in the development of closure expressions. The resulting model is then compared with measurements in several shear flows and satisfactory agreement between calculation and experiment is obtained.

(2) Intermittency factor

The equation for the intermittency factor [5],[8]

$$\frac{\partial \gamma}{\partial t} + \bar{U}_\alpha \frac{\partial \gamma}{\partial x_\alpha} = -\frac{\partial}{\partial x_\alpha}\left[\gamma(1-\gamma)(\bar{\bar{U}}_\alpha - \tilde{\tilde{U}}_\alpha)\right] + S_\gamma \qquad (1)$$

contains the rate of growth S_γ of the turbulent zone at the expense of the nonturbulent zone. In ref. [8],[9] several representations of S_γ

$$S_\gamma \equiv \langle V\delta^*(S)\rangle \tag{2}$$

were discussed. Note that V denotes the relative progression velocity of the interface and $\delta^*(S)$ is defined by

$$\delta^*(S) = |\nabla S|\delta(S) \tag{3}$$

with $S=0$ being the implicit equation for the interface (see [5],[8]). Since no representation of S_γ in terms of first and second order moments only is available, a closure model is required in the context of second order closures. Several closures have been suggested [4],[7],[10] for the source S_γ. We follow here the model suggested in ref. [7].

$$S_\gamma \cong -C_{g1}\gamma(1-\gamma)\frac{\widetilde{v_\alpha^* v_\beta^*}}{\tilde{k}}\left(\frac{\partial\tilde{v}_\alpha}{\partial x_\beta}+\frac{\partial\tilde{v}_\beta}{\partial x_\alpha}\right)+C_{g2}\frac{\tilde{k}^2}{\tilde{\varepsilon}}\frac{\partial\gamma}{\partial x_\alpha}\frac{\partial\gamma}{\partial x_\alpha}$$

$$-C_{g3}\gamma(1-\gamma)\frac{\tilde{\varepsilon}}{\tilde{k}} \tag{4}$$

with $C_{g1} = 1.8$, $C_{g2} = 0.15$, $C_{g3} = 0.05$. The first part in (4) represents growth of γ due to production of apparent stress in the turbulent zone. The second term on the right hand side of (4) reflects the transport of mass and momentum due to spatial inhomogeneity thus increasing the intermittency factor. The last term in (4) destructive, which leads to a decrease of γ in the absence of any production. Hobbs [11] observed in a wake flow without significant mean velocity gradients

that γ in fact decreases with downstream distance. Subsequent distortion of the wake flow lead to production of turbulence and immediate increase of intermittency factor [11]. Thus is S_γ expected to depend on the mean strain rate as suggested in (4).

The difference of the mean velocities in the turbulent and nonturbulent zones appearing in the intermittency equation (1) is in fact the turbulent diffusion of γ . This difference for the cross-flow component $(\alpha = 2)$ is for parabolic flows estimated by

$$\tilde{\bar{v}}_\alpha - \tilde{\tilde{v}}_\alpha \simeq C_g \frac{\tilde{\bar{k}}}{\tilde{\bar{\varepsilon}}} \overline{v_\alpha^* v_\beta^*} \frac{1}{\gamma} \frac{\partial \gamma}{\partial x_\beta} \qquad (C_g = 0.16) \qquad (5)$$

because of numerical inaccuracy in the calculation of the cross-flow component of $\tilde{\bar{v}}_\alpha$, $\tilde{\tilde{v}}_\alpha$ in boundary-layer-type flows. For elliptic flows are all momentum balances included in the system of equations and (5) would be avoided.

<u>(3) Mean velocity in turbulent and nonturbulent zones</u>

The exact equation for the turbulent zone mean velocity $\tilde{\bar{v}}_\alpha$

$$\frac{\partial \tilde{\bar{v}}_\alpha}{\partial t} + \bar{v}_\beta \frac{\partial \tilde{\bar{v}}_\alpha}{\partial x_\beta} = -\frac{1}{\gamma}\frac{\partial}{\partial x_\beta}\left(\gamma \overline{v_\alpha^* v_\beta^*}\right) + \nu \frac{\partial^2 \tilde{\bar{v}}_\alpha}{\partial x_\beta \partial x_\beta}$$

$$-\frac{1}{\rho}\frac{\partial \tilde{\bar{p}}}{\partial x_\alpha} + (1-\gamma)(\tilde{\tilde{v}}_\beta - \tilde{\bar{v}}_\beta)\frac{\partial \tilde{\bar{v}}_\alpha}{\partial x_\beta} + \frac{1}{\gamma} S_\alpha^* \qquad (6)$$

and for the nonturbulent zone $\tilde{\tilde{v}}_\alpha$

$$\frac{\partial \tilde{\tilde{v}}_\alpha}{\partial t} + \bar{v}_\beta \frac{\partial \tilde{\tilde{v}}_\alpha}{\partial x_\beta} = -\frac{1}{1-\gamma}\frac{\partial}{\partial x_\beta}\left[(1-\gamma)\overline{v_\alpha^o v_\beta^o}\right] + \nu \frac{\partial^2 \tilde{\tilde{v}}_\alpha}{\partial x_\beta \partial x_\beta}$$

$$-\frac{1}{\rho}\frac{\partial \tilde{\tilde{p}}}{\partial x_\alpha} - \gamma(\tilde{\tilde{v}}_\beta - \tilde{\bar{v}}_\beta)\frac{\partial \tilde{\tilde{v}}_\alpha}{\partial x_\beta} - \frac{1}{1-\gamma} S_\alpha^o \qquad (7)$$

contain interface terms S_α^* and S_α^o . They are linked by the consistency condition [7],[8]

$$S_\alpha^* - S_\alpha^o = (\tilde{\bar{u}}_\alpha - \bar{\bar{u}}_\alpha)(S_\gamma - \nu\Delta\gamma) - 2\nu\frac{\partial\gamma}{\partial x_\beta}\frac{\partial}{\partial x_\beta}(\tilde{\bar{u}}_\alpha - \bar{\bar{u}}_\alpha)$$

$$+ \frac{1}{\rho}(\tilde{\bar{p}} - \bar{\bar{p}})\frac{\partial\gamma}{\partial x_\alpha} \qquad (8)$$

for high Re-number are the viscous terms in (8) negligible and we obtain

$$S_\alpha^* - S_\alpha^o = (\tilde{\bar{u}}_\alpha - \bar{\bar{u}}_\alpha)\, S_\gamma + \frac{1}{\rho}(\tilde{\bar{p}} - \bar{\bar{p}})\frac{\partial\gamma}{\partial x_\alpha} \qquad (9)$$

Hence only one of the momentum sources S_α^* or S_α^o requires closure. It is helpful for the construction of a closure to consider the nonturbulent zone equation (7). The apparent stress term in (7) can be recast as follows

$$\frac{1}{1-\gamma}\frac{\partial}{\partial x_\beta}\left[(1-\gamma)\widetilde{\overline{u_\alpha^o u_\beta^o}}\right] = \frac{\partial}{\partial x_\beta}\widetilde{\overline{u_\alpha^o u_\beta^o}} - \frac{\widetilde{\overline{u_\alpha^o u_\beta^o}}}{1-\gamma}\frac{\partial\gamma}{\partial x_\beta} \qquad (10)$$

The first part has the proper divergence form of a diffusive term whereas the second part is a source term. Comparing (10) with the corresponding expression in the turbulent zone (eq. (16)

$$\frac{1}{\gamma}\frac{\partial}{\partial x_\beta}\left(\gamma\,\overline{\overline{u_\alpha^* u_\beta^*}}\right) = \frac{\partial}{\partial x_\beta}\overline{\overline{u_\alpha^* u_\beta^*}} + \frac{1}{\gamma}\overline{\overline{u_\alpha^* u_\beta^*}}\frac{\partial\gamma}{\partial x_\beta}$$

a similar source term but with the opposite sign appears. Considering a shear flow such as a jet, it becomes clear that for the turbulent zone is the second part is a genuine source, whereas for the nonturbulent zone it is a sink, which leads to wave-like variations of $\tilde{\bar{u}}_\alpha$ if included in the equation (7). Since there is no experimental or theoretical evidence

for such solutions, this sink in (7) must be balanced by a corresponding term in S_α^o. Furthermore is momentum transfer between turbulent and nonturbulent zones only possible in the mean if $\bar{\tilde{v}}_\alpha$ and $\tilde{\bar{v}}_\alpha$ are not equal. Hence the following closure model for S_α^o is suggested

$$S_\alpha^o \cong C_4\gamma(1-\gamma)\frac{\bar{\tilde{\varepsilon}}}{\bar{\tilde{k}}}(\tilde{\bar{v}}_\alpha - \bar{\tilde{v}}_\alpha) + \widetilde{\overline{v_\alpha^o v_\beta^o}}\frac{\partial\gamma}{\partial x_\beta} \qquad (11)$$

with $C_4 = 1.0$. Note that S_α^o vanishes, if $\tilde{\bar{v}}_\alpha = \bar{\tilde{v}}_\alpha$ for a region with finite volume, which implies that $\partial\gamma/\partial x_\beta$ approaches zero also. The consistency condition (9) yields then

$$S_\alpha^* \cong (\tilde{\bar{v}}_\alpha - \bar{\tilde{v}}_\alpha)\left[C_4\gamma(1-\gamma)\frac{\bar{\tilde{\varepsilon}}}{\bar{\tilde{k}}} + S_\gamma\right]$$

$$+ \widetilde{\overline{v_\alpha^o v_\beta^o}}\frac{\partial\gamma}{\partial x_\beta} + \frac{1}{\rho}(\tilde{\bar{p}} - \bar{\tilde{p}})\frac{\partial\gamma}{\partial x_\alpha} \qquad (12)$$

Note that for the longitudinal component $(\alpha=1)$ the pressure term in (12) becomes negligible. The apparent stresses in both zones are included in the system of variables and therefore conclude (11) and (12) the closure of the mean velocity equation.

(4) Stresses in turbulent and nonturbulent zones

The exact transport equations for the apparent stresses in turbulent and nonturbulent zones follow from mass and momentum balances [9]. They can be given for high Re-numbers in the form

$$\frac{\partial}{\partial t}\overline{\overline{v_\alpha^* v_\beta^*}} + \bar{\tilde{v}}_\gamma\frac{\partial}{\partial x_\gamma}\overline{\overline{v_\alpha^* v_\beta^*}} = -\overline{\overline{v_\alpha^* v_\gamma^*}}\frac{\partial\bar{\tilde{v}}_\beta}{\partial x_\gamma} - \overline{\overline{v_\beta^* v_\gamma^*}}\frac{\partial\bar{\tilde{v}}_\alpha}{\partial x_\gamma} - \bar{\tilde{\varepsilon}}_{\alpha\beta}$$

$$+ \frac{1}{\gamma}\frac{\partial}{\partial x_\gamma}(\gamma\,\mathcal{D}_{\alpha\beta\gamma}^*) + \frac{1}{\rho}\overline{\overline{p^*\left(\frac{\partial v_\alpha^*}{\partial x_\beta} + \frac{\partial v_\beta^*}{\partial x_\alpha}\right)}} - \frac{1}{\gamma}S_\gamma\overline{\overline{v_\alpha^* v_\beta^*}} + \frac{1}{\gamma}S_{\alpha\beta}^* \qquad (13)$$

and

$$\frac{\partial}{\partial t}\widetilde{v_\alpha^\circ v_\beta^\circ} + \tilde{\bar{v}}_\gamma \frac{\partial}{\partial x_\gamma}\widetilde{v_\alpha^\circ v_\beta^\circ} = -\widetilde{v_\alpha^\circ v_\gamma^\circ}\frac{\partial \tilde{\bar{v}}_\beta}{\partial x_\gamma} - \widetilde{v_\beta^\circ v_\gamma^\circ}\frac{\partial \tilde{\bar{v}}_\alpha}{\partial x_\gamma} - \tilde{\bar{\varepsilon}}_{\alpha\beta}$$

$$+\frac{1}{1-\gamma}\frac{\partial}{\partial x_\gamma}\left[(1-\gamma)\mathcal{D}^\circ_{\alpha\beta\gamma}\right] + \frac{1}{\tilde{\bar{\rho}}}\overline{p^\circ\left(\frac{\partial v_\alpha^\circ}{\partial x_\beta} + \frac{\partial v_\beta^\circ}{\partial x_\alpha}\right)} + \frac{1}{1-\gamma}S_\gamma \widetilde{v_\alpha^\circ v_\beta^\circ} - \frac{1}{1-\gamma}S^\circ_{\alpha\beta} \qquad (14)$$

The diffusive fluxes are defined by [9]

$$\mathcal{D}^*_{\alpha\beta\gamma} = -\overline{\widetilde{v_\alpha^* v_\beta^* v_\gamma^*}} - \delta_{\alpha\gamma}\overline{\widetilde{v_\beta^* p^*}} - \delta_{\beta\gamma}\overline{\widetilde{v_\alpha^* p^*}} \qquad (15)$$

and

$$\mathcal{D}^\circ_{\alpha\beta\gamma} = -\widetilde{v_\alpha^\circ v_\beta^\circ v_\gamma^\circ} - \delta_{\alpha\gamma}\widetilde{v_\beta^\circ p^\circ} - \delta_{\beta\gamma}\widetilde{v_\alpha^\circ p^\circ} \qquad (16)$$

The interface terms $S^*_{\alpha\beta}$ and $S^\circ_{\alpha\beta}$ are defined in ref. [9] and satisfy the consistency relation (see Eq. (42) in [9])

$$S^*_{\alpha\beta} - S^\circ_{\alpha\beta} = (1-2\gamma)\Delta v_\alpha \Delta v_\beta S_\gamma - \gamma\left[\Delta v_\alpha S_\beta^\circ + \Delta v_\beta S_\alpha^\circ\right]$$

$$-(1-\gamma)\left[\Delta v_\alpha S_\beta^* + \Delta v_\beta S_\alpha^*\right] + (1-\gamma)\widetilde{v_\alpha^\circ v_\beta^\circ}\frac{\partial}{\partial x_\gamma}(\gamma\Delta v_\gamma)$$

$$-\gamma\,\overline{\widetilde{v_\alpha^* v_\beta^*}}\frac{\partial}{\partial x_\gamma}\left[(1-\gamma)\Delta v_\gamma\right] \qquad (17)$$

where $\Delta v_\alpha \equiv \tilde{\bar{v}}_\alpha - \tilde{\bar{\bar{v}}}_\alpha$.

This relation (17) is valid for high Re-numbers and $\Delta p \equiv \tilde{\bar{p}} - \bar{\bar{p}} = 0$.

It is reasonable (at least for thin shear layers) to assume that the mean pressures in the two zones are equal because $\tilde{\bar{p}} \to \bar{p}$ for $\gamma \to 0$

(outer edge) and $\tilde{\bar{\rho}} \to \bar{\rho}$ for $\gamma \to 1$ (center of shear layer) and $\bar{\rho}$ is constant across a thin shear layers in first order approximation. Hence is $\bar{\rho} = \hat{\bar{\rho}} = \tilde{\bar{\rho}}$ a good approximation to first order.

The closure of the stress equations is concerned with two groups of terms: The classical terms, which are counterparts of terms in the unconditional stress equations, and the interface terms. For the closure of the classical term group existing models [3],[12],[7] are carried over from the unconditional to the conditional correlations. This is certainly justified, because closure assumptions based on quasi-Gaussian behaviour of higher moments or relaxation to Gaussianity in the absence of strain rates and boundaries (such as Lumley's diffusion closure [12]) hold with better accuracy for conditioned moments than for unconditioned moments. This follows from the observation, that conditioning removes the spike in the pdf corresponding to the other (nonturbulent) zone thus bringing in particular flatness factors closer to the Gaussian values. This is backed up by experimental results [13],[14]. The interface terms on the other hand require new considerations.

4.1. Closure of the turbulent-zone-stress equations

The closure of the classical terms requires only brief discussion, because established closure models will be modified for conditioned correlations. The dissipation of stress is taken in high Re-number form [3]

$$\tilde{\bar{\varepsilon}}_{\alpha\beta} \hat{=} \frac{2}{3} \delta_{\alpha\beta} \tilde{\bar{\varepsilon}} \tag{18}$$

where the dissipation rate $\tilde{\bar{\varepsilon}}$ is given by

$$\tilde{\bar{\varepsilon}} = \frac{1}{2} \tilde{\bar{\varepsilon}}_{\alpha\alpha}$$

For the diffusive flux $\mathscr{D}^*_{\alpha\beta\gamma}$ Lumley's model [12] for the triple correlations is applied

$$\mathscr{D}^*_{\alpha\beta\gamma} \cong \frac{2}{3C_L} \frac{\tilde{\tilde{k}}}{\tilde{\tilde{\varepsilon}}} \left[\tilde{\tilde{\mathscr{D}}}_{\alpha\beta\gamma} + \frac{C_L - 2}{2C_L + 5} \left(\delta_{\alpha\beta} \tilde{\tilde{\mathscr{D}}}_\gamma + \delta_{\alpha\gamma} \tilde{\tilde{\mathscr{D}}}_\beta + \delta_{\beta\gamma} \tilde{\tilde{\mathscr{D}}}_\alpha \right) \right] \quad (19)$$

where

$$\tilde{\tilde{\mathscr{D}}}_{\alpha\beta\gamma} \equiv \widetilde{\widetilde{v^*_\alpha v^*_\eta}} \frac{\partial}{\partial x_\eta} \widetilde{\widetilde{v^*_\beta v^*_\gamma}} + \widetilde{\widetilde{v^*_\beta v^*_\eta}} \frac{\partial}{\partial x_\eta} \widetilde{\widetilde{v^*_\alpha v^*_\gamma}} + \widetilde{\widetilde{v^*_\gamma v^*_\eta}} \frac{\partial}{\partial x_\eta} \widetilde{\widetilde{v^*_\alpha v^*_\beta}} \quad (20)$$

and

$$\tilde{\tilde{\mathscr{D}}}_\alpha \equiv \widetilde{\widetilde{v^*_\alpha v^*_\eta}} \frac{\partial \tilde{\tilde{k}}}{\partial x_\eta} + \widetilde{\widetilde{v^*_\gamma v^*_\eta}} \frac{\partial}{\partial x_\eta} \widetilde{\widetilde{v^*_\alpha v^*_\gamma}} \quad (21)$$

The constant C_L was set to $C_L = 7.5$. The pressure correlations in $\mathscr{D}^*_{\alpha\beta\gamma}$ are neglected. This model is based on the notion that turbulence relaxes to Gaussian statistics for the large scales if inhomogeneities are removed. It is therefore better suited for conditioned than for *un*conditioned moments. The pressure-rate of strain correlation in (13) are modelled according to Rotta [15] ("return to isotropy") and Hanjalic and Launder [16]

$$\frac{1}{\rho} \widetilde{\widetilde{p^* \left(\frac{\partial v^*_\alpha}{\partial x_\beta} + \frac{\partial v^*_\beta}{\partial x_\alpha} \right)}} \cong - C_1 \frac{\tilde{\tilde{\varepsilon}}}{\tilde{\tilde{k}}} \left(\widetilde{\widetilde{v^*_\alpha v^*_\beta}} - \frac{2}{3} \delta_{\alpha\beta} \tilde{\tilde{k}} \right)$$

$$- \frac{C_2 + 8}{11} \left(P_{\alpha\beta} - \frac{2}{3} \delta_{\alpha\beta} P \right) - \frac{30 C_2 - 2}{55} \tilde{\tilde{k}} \left(\frac{\partial \tilde{\tilde{v}}_\alpha}{\partial x_\beta} + \frac{\partial \tilde{\tilde{v}}_\beta}{\partial x_\alpha} \right) - \frac{8 C_2 - 2}{11} \left(D_{\alpha\beta} - \frac{2}{3} \delta_{\alpha\beta} P \right) \quad (22)$$

where

$$P_{\alpha\beta} \equiv - \widetilde{\widetilde{v^*_\alpha v^*_\gamma}} \frac{\partial \tilde{\tilde{v}}_\beta}{\partial x_\gamma} - \widetilde{\widetilde{v^*_\beta v^*_\gamma}} \frac{\partial \tilde{\tilde{v}}_\alpha}{\partial x_\gamma} \quad , \quad P = \frac{1}{2} P_{\alpha\alpha} \quad (23)$$

and

$$D_{\alpha\beta} \equiv - \widetilde{\widetilde{v^*_\alpha v^*_\gamma}} \frac{\partial \tilde{\tilde{v}}_\gamma}{\partial x_\beta} - \widetilde{\widetilde{v^*_\beta v^*_\gamma}} \frac{\partial \tilde{\tilde{v}}_\gamma}{\partial x_\alpha} \quad (24)$$

with

$$C_1 = 1.5$$
$$C_2 = 0.4$$

The closure of the interface terms $S^*_{\alpha\beta}$ is based on the following considerations. The turbulent zone propagates into the nonturbulent zone by viscous transport of vorticity into irrotational parcels of fluid. This propagation is only possible if the net effect on the turbulent zone stress is gain at the expense of nonturbulent zone fluctuations and if the fluctuations in the turbulent zone are weaker than in the nonturbulent zone. Thus the model

$$S^*_{\alpha\beta} \cong C_5 \gamma (1-\gamma)\left(\widetilde{\overline{u^\circ_\alpha u^\circ_\beta}} - \widetilde{\overline{u^*_\alpha u^*_\beta}}\right)\frac{\bar{\bar{\varepsilon}}}{\bar{\bar{k}}} \tag{25}$$

emerges with $C_5 = 0.7$. Thus the closure of the turbulent zone stress equations is concluded. The constants for the classical terms (C_L, C_1, C_2) are taken from the respective references and the constant for the interface group $S^*_{\alpha\beta}$ was established by computer optimization.

4.2. Closure of the nonturbulent-zone-stress equations

The closure for diffusive flux and pressure correlations for the nonturbulent zone cannot be simply carried over from the unconditional case, because the nonturbulent zone fluctuations behave differently (they are irrotational) in the limit $\gamma \to 0$ from their turbulent zone counterparts. Since no homogeneous distribution exists in the nonturbulent zone, the decay properties of correlations in this zone with distance from the turbulent zone as presented in [9] are used to estimate the relative order of magnitude of the terms in (14). Analysis of the stress equations shows that for $\gamma \to 0$ diffusion and pressure strain

rate correlation become the leading terms. Furthermore is for this limit the normal stress component $\widetilde{\overline{v_2^{o2}}}$ dominant. In order to take the decay of the fluctuations with distance from the turbulent zone into account the following closure expressions is used with a time scale τ_{NT} for the nonturbulent zone

$$\frac{\partial}{\partial x_2}\left(\tau_{NT}\,\widetilde{\overline{v_2^{o2}}}\,\frac{\partial \widetilde{\overline{v_2^{o2}}}}{\partial x_2}\right) \sim \frac{1}{\tau_{NT}}\left[\widetilde{\overline{v_2^{o2}}} - \frac{2}{3}\tilde{k}\right]$$

and with the decay law [17],[9]

$$\widetilde{\overline{v_2^{o2}}} \sim y^{-4}$$

where y denotes the normal distance from the center of the turbulent zone, we obtain for

$$\tau_{NT} = \tau_T\, y^{n} \quad , \quad \tau_T = \frac{\tilde{\bar{k}}}{\tilde{\bar{\varepsilon}}}$$

where τ_T denotes the turbulent zone scale, the decay law

$$\tau_{NT} \sim y^{3}$$

Let furthermore γ be approximated by

$$\gamma \sim y^{-\kappa}$$

for large y , then follows

$$\tau_{NT} = \frac{\tau_T}{\gamma} = \frac{\tilde{\bar{k}}}{\gamma\,\tilde{\bar{\varepsilon}}} \qquad (26)$$

for $n = \kappa = 3$. This consideration indicates the method of closure for the nonturbulent zone. The classical terms will expressed in terms of the unconditional closure with the modified time scale τ_{NT} given by (26).

The dissipation in (14) is negligible, because viscosity has no effect on momentum transport in irrotational flow. The closure for the diffusive flux is given by [12]

$$\mathcal{D}^{o}_{\alpha\beta\gamma} \cong \frac{2}{3C_L}\frac{\tilde{\bar{k}}}{\gamma\tilde{\bar{\varepsilon}}}\left[\tilde{\mathcal{D}}_{\alpha\beta\gamma} + \frac{C_L-2}{2C_L+5}\left(\delta_{\alpha\beta}\tilde{\mathcal{D}}_{\gamma} + \delta_{\alpha\gamma}\tilde{\mathcal{D}}_{\beta} + \delta_{\beta\gamma}\tilde{\mathcal{D}}_{\alpha}\right)\right] \quad (27)$$

where

$$\tilde{\mathcal{D}}_{\alpha\beta\gamma} = \widetilde{\overline{v^o_\alpha v^o_\eta}}\frac{\partial}{\partial x_\eta}\widetilde{\overline{v^o_\beta v^o_\gamma}} + \widetilde{\overline{v^o_\beta v^o_\eta}}\frac{\partial}{\partial x_\eta}\widetilde{\overline{v^o_\alpha v^o_\gamma}} + \widetilde{\overline{v^o_\gamma v^o_\eta}}\frac{\partial}{\partial x_\eta}\widetilde{\overline{v^o_\alpha v^o_\beta}} \quad (28)$$

and

$$\tilde{\mathcal{D}}_{\alpha} = \widetilde{\overline{v^o_\alpha v^o_\eta}}\frac{\partial \tilde{\bar{k}}}{\partial x_\eta} + \widetilde{\overline{v^o_\gamma v^o_\eta}}\frac{\partial}{\partial x_\eta}\widetilde{\overline{v^o_\alpha v^o_\gamma}} \quad (29)$$

and C_L has the same value as for the turbulent zone. The pressure-strain rate correlations are again modelled as in the unconditioned case with the modified time scale τ_{NT} :

$$\frac{1}{\rho}\widetilde{\overline{p^o\left(\frac{\partial v^o_\alpha}{\partial x_\beta} + \frac{\partial v^o_\beta}{\partial x_\alpha}\right)}} \cong -C_1(2-\gamma)\frac{\gamma\tilde{\bar{\varepsilon}}}{\tilde{\bar{k}}}\left(\widetilde{\overline{v^o_\alpha v^o_\beta}} - \frac{2}{3}\delta_{\alpha\beta}\tilde{\bar{k}}\right)$$

$$-\gamma\left[\frac{C_2+8}{11}\left(P_{\alpha\beta} - \frac{2}{3}\delta_{\alpha\beta}P\right) + \frac{30C_2-2}{55}\tilde{k}\left(\frac{\partial\tilde{\bar{U}}_\alpha}{\partial x_\beta} + \frac{\partial\tilde{\bar{U}}_\beta}{\partial x_\alpha}\right) + \frac{8C_2-2}{11}\left(D_{\alpha\beta} - \frac{2}{3}\delta_{\alpha\beta}P\right)\right] \quad (30)$$

The definitions of $P_{\alpha\beta}$, P, $D_{\alpha\beta}$ are as in (23) and (24) except nonturbulent zone quantities and averages replace the turbulent zone symbols. The second part in (30) representing the "rapid" terms is multiplied with the intermittency factor to modify the time scale determined by the strain rate. The first part in (30) has an additional variation with γ in terms of the factor $(2-\gamma)$ in order to compensate for the decrease with γ of the second ("rapid") part. Both

constants C_1 and C_2 have the same values as for the turbulent zone counterparts (22)-(24).

The closure of the interface group $S^o_{\alpha\beta}$ is guided by the properties of the solution induced by the second part of

$$\frac{1}{1-\gamma}\frac{\partial}{\partial x_\delta}\left[(1-\gamma)\mathcal{D}^o_{\alpha\beta\delta}\right] = \frac{\partial}{\partial x_\delta}\mathcal{D}^o_{\alpha\beta\delta} - \frac{\mathcal{D}^o_{\alpha\beta\delta}}{1-\gamma}\frac{\partial\gamma}{\partial x_\delta}$$

and

$$\frac{1}{1-\gamma}S_\gamma \widetilde{v^o_\alpha v^o_\beta}$$

Both lead to wavy solutions and thus the model

$$S^o_{\alpha\beta} \cong S^*_{\alpha\beta} + \mathcal{D}^o_{\alpha\beta\delta}\frac{\partial\gamma}{\partial x_\delta} + 2\widetilde{v^o_\alpha v^o_\beta}\, S_\gamma \tag{31}$$

is suggested. It is a preliminary form, which gives well-behaved solutions, but different forms using the consistency condition (17) are currently being investigated. Thus the closure of the nonturbulent zone stress equations is concluded without new constants.

4.3. Dissipation rate

The dissipation rate $\bar{\tilde{\varepsilon}}$ in the turbulent zone satisfies a complex equation [16] with all sink/source terms and turbulent diffusion in non-closed form. The closed form follows Hanjalic and Launder [16] except for the turbulent diffusion which is taken from Lumley [12] and the interface group

$$S_\varepsilon \cong C_6(1-\gamma)\frac{\bar{\tilde{\varepsilon}}^2}{\bar{\bar{k}}^2}(\tilde{\bar{k}} - \bar{\tilde{k}}) \tag{32}$$

with $C_6 = 0.7$ determined by computer optimization. Thus the following equation is obtained:

$$\frac{\partial \tilde{\varepsilon}}{\partial t} + \tilde{u}_\alpha \frac{\partial \tilde{\varepsilon}}{\partial x_\alpha} = -\frac{1}{\gamma}\frac{\partial}{\partial x_\alpha}\left(\gamma\, \overline{\widetilde{u_\alpha^* \varepsilon^*}}\right) - C_{\varepsilon 1}\frac{\tilde{\varepsilon}}{\tilde{k}}\, \overline{\widetilde{u_\alpha^* u_\beta^*}}\, \frac{\partial \tilde{u}_\alpha}{\partial x_\beta}$$

$$- C_{\varepsilon 2}\frac{\tilde{\varepsilon}^2}{\tilde{k}} - \frac{1}{\gamma} S_\gamma \tilde{\varepsilon} + C_6 (1-\gamma)\frac{\tilde{\varepsilon}^2}{\tilde{k}^2}\left(\tilde{k} - \bar{k}\right) \quad (33)$$

where [12]

$$-\overline{\widetilde{u_\alpha^* \varepsilon^*}} \triangleq C_\varepsilon \frac{\tilde{k}}{\tilde{\varepsilon}}\left(\overline{\widetilde{u_\alpha^* u_\beta^*}} + \frac{1}{\tilde{k}}\, \overline{\widetilde{u_\gamma^* u_\beta^*}} \cdot \overline{\widetilde{u_\alpha^* u_\gamma^*}}\right)\frac{\partial \tilde{\varepsilon}}{\partial x_\beta} \quad (34)$$

The constants are $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 2.0$, $C_\varepsilon = 0.15$.

(5) Numerical Solution

The system of partial differential equations constituting the conditional closure model reduce to parabolic form for boundary-layer-type flows. Thus they can be solved in marching type integration. The present method is the standard finite-difference procedure developed by Patankar and Spalding [18]. The new aspect of the solution method is the introduction of a block-solver [19] for the turbulent and nonturbulent zone stress tensors. The reason for this is the strong coupling of the stresses as a consequence of the diffusion models (19) and (27) which makes sequential solution likely to be unstable. Thus are the stress equations (13) and (14) respectively solved simultaneously forming two block-tri-diagonal systems of equations with blocksize $N_B = 4$ in both cases. All other equations are solved sequentially in the marching integration step.

The discretization of the differential equation was performed using staggered grids. All first order moments (intermittency factor and mean velocities) were defined at node points, whereas all second order moments were defined at the midpoints. This leads to nearly second order accurate representation of production terms in the stress equations and the diffusive term in the mean velocity equations. Furthermore is the stability of the finite-difference scheme improved [7].

The initial conditions for conditional closure models can be set in several ways. The turbulent zone variables and the intermittency factor can be prescribed corresponding to fully turbulent flow or the intermittency factor is started with a small value and the nonturbulent variables with laminar profiles representing a slightly disturbed laminar flow. In the present case the former method was chosen.

The boundary conditions for the apparent stress tensor in the nonturbulent zone require some consideration. Since the nonturbulent zone stresses cannot approach a homogeneous distribution the decay laws discussed in ref. [9] are used to establish $\overline{\tilde{v_k^\circ v_k^\circ}}$ at the free boundary. This is done in terms of a gradient condition

$$\frac{\partial}{\partial y} \overline{\tilde{v_k^\circ v_k^\circ}} \simeq -\frac{4}{y-y_0} \overline{\tilde{v_k^\circ v_k^\circ}} \quad for \quad y \gg y_0$$

where y is the normal distance from the center of the turbulent region (symmetry axis on location of maximal shear stress in turbulent zone). The location of the origin y_0 for y is not known exactly, but for large values of y this relation becomes reasonably accurate.

All calculations were performed with $N = 50$ grid points over the cross-section and the number of steps in x-direction ranged from 800 to 2000 depending on the length of the computational domain.

(6) Applications

The conditional second order closure developed above was applied to the calculation of several plane shear layers. All calculations were carried out with the same set of constants given in chapters 2. to 4.

6.1. Plane jet

The results for the plane jet are shown in fig. 1. to fig. 10. in the nearly self-similar region. The experiments of Gutmark and Wygnanski [20] and Sunyach [21] are used for comparison with the calculations. The intermittency factor profile in fig. 1 lies between the two sets of measurements (open symbols: [20], full symbols [21]), but is somewhat steeper than the experimental profile. The mean velocity for the turbulent zone, the nonturbulent zone and unconditioned are compared with the experiment [20] in fig. 2. The nonturbulent zone mean velocity $\tilde{\tilde{U}}$ in fig. 2 and in fig. 3 is higher than the turbulent zone mean at the axis, because the nonturbulent zone shear stress (fig. 4) is always less than its turbulent zone counterpart thus leading to slower decay of $\tilde{\tilde{U}}$. The calculated shear stress (conditioned and unconditioned) in fig. 4 is lower than the experiment [20] in the outer part of the flow, but agrees reasonably well with the measurements in the main part of the flow field. Consequently is the spreading rate $dy_{0.5}/dx \cong 0.108$ close to the experimental value $dy_{0.5}/dx \cong 0.11$. The normal stress components are shown in fig. 5 to fig. 10. Their relative magnitude can be evaluated from fig. 5 for the unconditioned case, from fig. 6 for the nonturbulent zone and from fig. 7 for the turbulent zone. It is clear

from fig. 6 that $\widetilde{\overline{v_2^{o2}}}$ becomes dominant as y increases, which is in accordance with the relation [9]

$$\widetilde{\overline{v_2^{o2}}} = \widetilde{\overline{v_1^{o2}}} + \widetilde{\overline{v_3^{o2}}}$$

as $y \to 0$. This property is due to Lumley's diffusion closure (27) and the interface terms (31). If the simpler gradient-flux model of Daly and Harlow [22] is applied this relation cannot be satisfied as the outer edge is approached. The comparison of the normal stress profiles with the available experimental data [20] shows reasonable agreement fig. 8 - fig. 10. In particular are the shapes of $\overline{\overline{v_2^{*2}}}$ and $\overline{v_2^{\prime 2}}$ well predicted.

6.2. Plane wake

The downstream region $(x/D = 200)$ for the plane wake of a cylinder is compared with the measurements of Fabris [23] and Thomas [24] in fig. 11 - 20. The profiles for the intermittency factor in fig. 11 is about ten percent less wide than in the experiments (open symbols: [23], full symbols [24]), but the slope agrees well with the results from [23]. The mean velocities in fig. 12 are in close agreement with the experiments [23]. The turbulent zone mean velocity $\bar{\bar{U}}$ shows a slight bulge near the outer edge, which is due to the boundary condition set to be equal to the free stream. The nonturbulent zone mean $\tilde{\bar{U}}$ in fig. 13 does not allow complete comparison because only limited data are available. The comparison of shear stresses fig. 14 and normal stresses fig. 15 to fig. 20 shows much the same properties as for the jet. It is noteworthy that $\overline{v_2^{\prime 2}}$ and $\overline{\overline{v_2^{*2}}}$ in fig. 19 are close to the measurements as for the jet.

6.3. Plane mixing layer

The plane mixing layer is a flow, which is rather difficult to predict, because of its sensitivity towards the flow conditions and the appearance of coherent structures [25], which is reflected in a wide range of observed spreading rates and significant variation in the experimental data [26]. The calculations for the conditions given in [27] are compared with the experiments in fig. 21 to fig. 80. The intermittency factor in fig. 21 shows a broader profile than the experiments of Wygnanski and Fiedler [27] but agrees on the high-speed side with [28] and is lower than the data of [29] on the low-speed side. The agreement of the mean velocities in fig. 22 with the experiments [27] is reasonable. The turbulent zone profile (full circles) from [27] is not approaching the free stream on the low speed side. It is not clear whether this is due to a different normalization of the experimental data on a genuine tendency. The comparison of calculated mean velocity for the nonturbulent zone with experiments [27] in fig. 23 is quite good and extends over the complete profile. The comparison of the stress components in fig. 24-30 with the experiments [27] is better on the low-speed side than on the high speed side. This can be traced back to the dissipation rate equation which does not produce the correct length scale profile. This could be improved by using a length scale which is constant over the cross-section.

(7) Conclusions

The closure of the first and second order moment equations for conditioned variables was developed and the resulting system of model equations was applied to the calculation of several shear flows. The following conclusions can be drawn from the properties of the model and

the comparison with available experimental data.

(1) The intermittency factor γ plays a central role in conditional closures, because all unconditioned moments are combinations of conditional moments and the intermittency factor. The source term of the equation for the intermittency factor requires closure, which is constructed as difference of production due to creation of apparent stress in the turbulent zone and the inhomogeneity of the γ-distribution and destruction due to viscous effects. The calculated γ-profile appears in all test cases in reasonable agreement with the measurements.

(2) The conditioned moment equations contain interface terms, that represent the transport of mass and momentum due to the movement of the interface. If it is assumed that the conditioned mean pressure in both zones is equal, then follows a local and closed consistency condition eliminating the need for closure of the interface terms in one of the zones. This is used to advantage for the mean velocity equations, but for the stress equation only a simplified version is applied. The results show that the nonturbulent zone mean velocity is in good agreement with the limited experimental data available. The comparison of the nonturbulent zone stresses is too incomplete to draw definite conclusions however.

(3) The notion of conditioning allows further application to reacting flows (in particular premixed flames [30]) and generalization to nonlocal conditions to deal with structural information not accessible to single-point theories.

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Figure captions

Fig. 1 Intermittency factor for the plane jet.

($\eta = y/y_{0.5}$, Symbols: O[20], ●[21]).

Fig. 2 Mean velocities for the plane jet.

(Symbols: [20]).

Fig. 3 Mean velocity in nonturbulent zone of the plane jet.

(Symbols: [20]).

Fig. 4 Shear stresses for the plane jet. (Symbols: [20]).

Fig. 5 Unconditioned normal stresses for the plane jet.

Fig. 6 Nonturbulent zone normal stresses for the plane jet.

Fig. 7 Turbulent zone normal stresses for the plane jet.

Fig. 8 Normal stresses in the plane jet compared with experiment [20].

Fig. 9 Normal stresses in the plane jet compared with experiment [20].

Fig. 10 Normal stresses in the plane jet compared with experiment [20].

Fig. 11 Intermittency factor for the plane wake

($\eta = y/y_{0.5}$, Symbols: O[23], ●[24]).

Fig. 12 Mean velocities for the plane wake (Symbols: [23]).

Fig. 13 Mean velocity in nonturbulent zone of the plane wake

(Symbols: [23]).

Fig. 14 Shear stresses for the plane wake (Symbols: [23]).

Fig. 15 Unconditioned normal stresses for the plane wake.

Fig. 16 Nonturbulent zone normal stresses for the plane wake.

Fig. 17 Turbulent zone normal stresses for the plane wake.

Fig. 18 Normal stresses in the plane wake compared with experiment [23].

Fig. 19 Normal stresses in the plane wake compared with experiment [23].

Fig. 20 Normal stresses in the plane wake compared with experiment [23].

Fig. 21 Intermittency factor for the plane mixing layer ($y=(Y-Y_{0.5})/(Y_{0.1}-Y_{0.9})$ Symbols: O[27], ●[28], □ [29]).

Fig. 22 Mean velocities for the plane mixing layer (Symbols: [27]).

Fig. 23 Mean velocity in nonturbulent zone for the plane mixing layer (Symbols: [27]).

Fig. 24 Shear stresses for the plane mixing layer (Symbols: [27]).

Fig. 25 Unconditioned normal stresses for the plane mixing layer.

Fig. 26 Nonturbulent zone normal stresses for the plane mixing layer.

Fig. 27 Turbulent zone normal stresses for the plane mixing layer.

Fig. 28 Normal stresses in the plane mixing layer compared with experiment [27].

Fig. 29 Normal stresses in the plane mixing layer compared with experiment [27].

Fig. 30 Normal stresses in the plane mixing layer compared with experiment [27].

Fig. 1

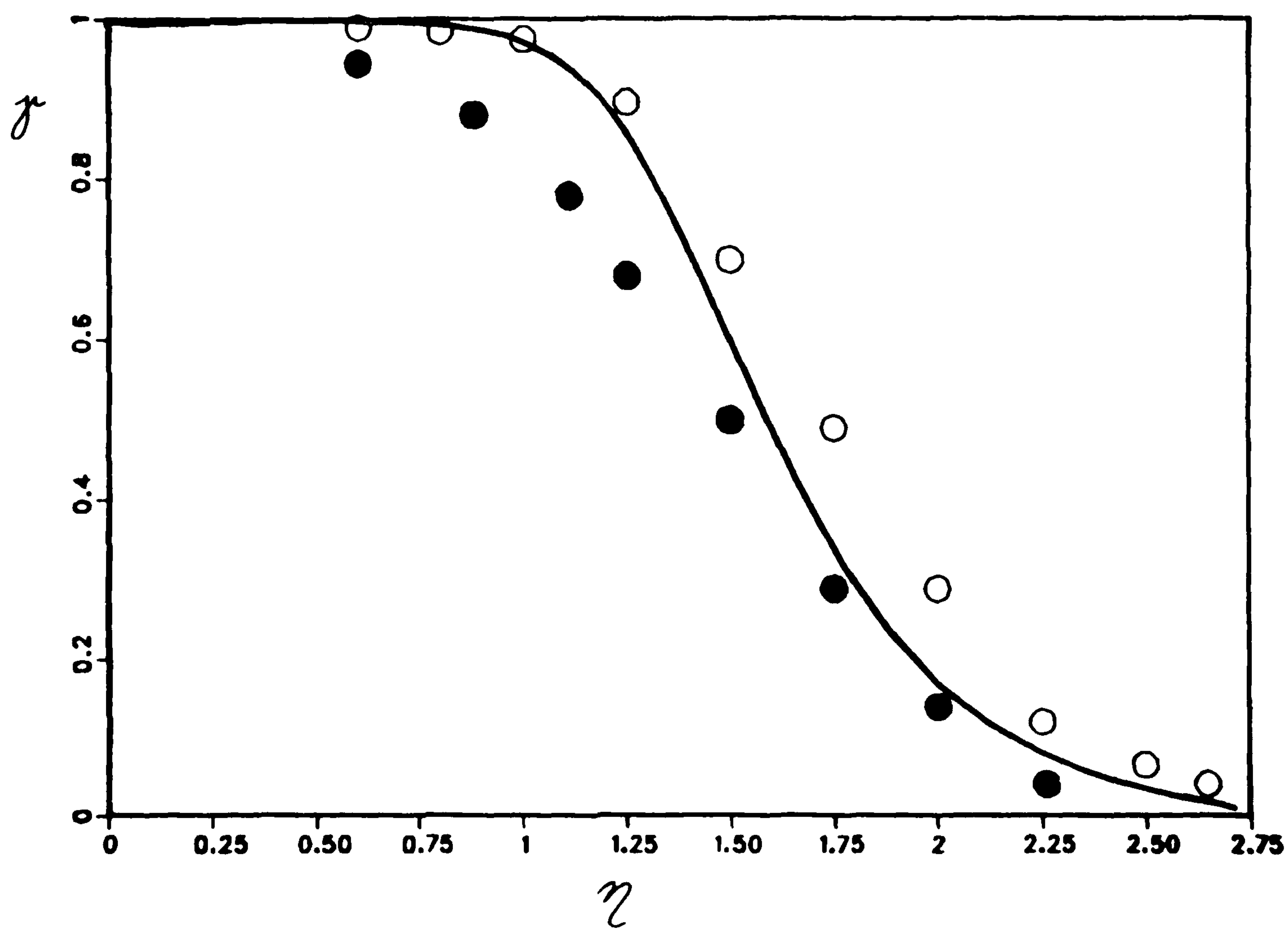
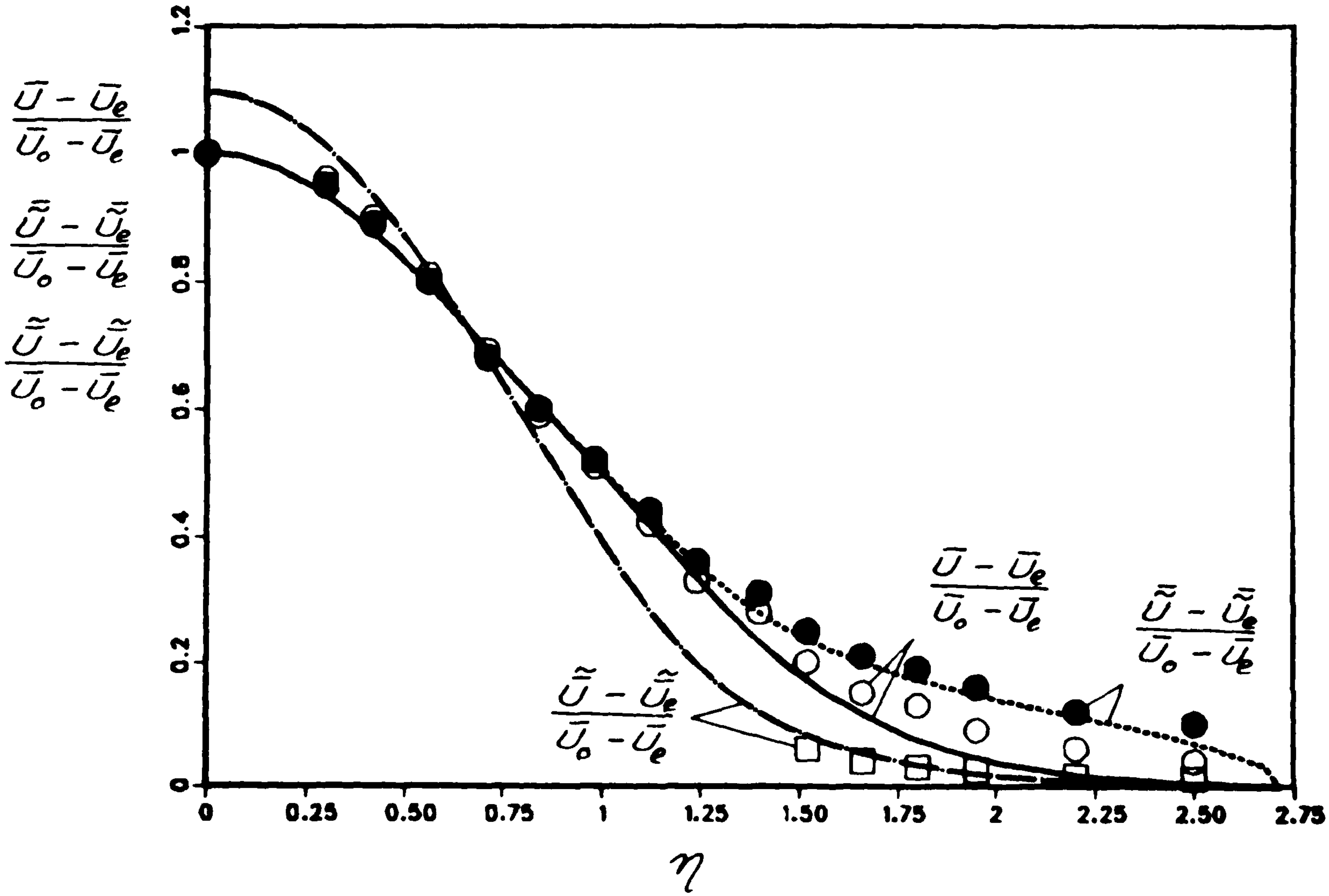
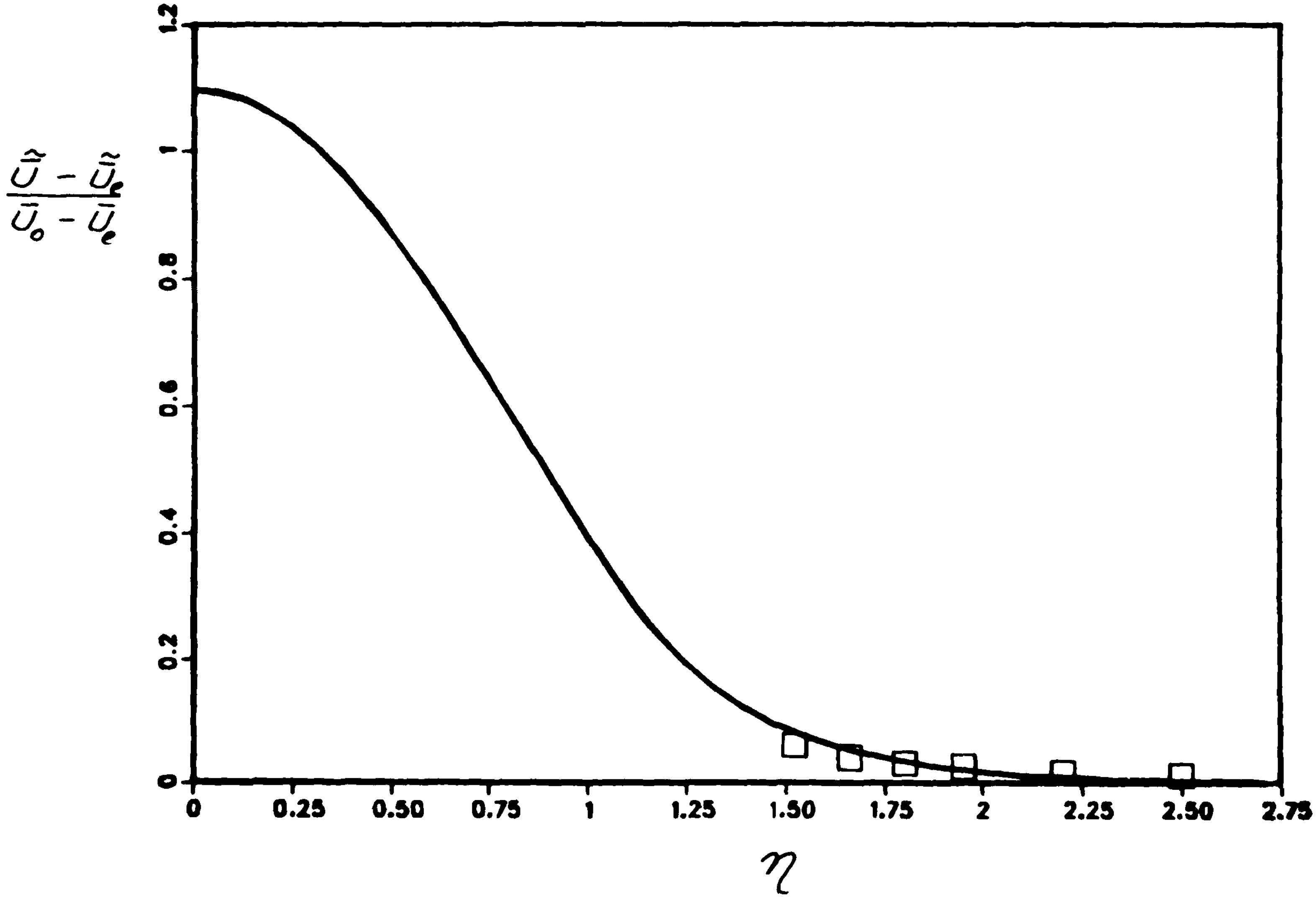


Fig. 2



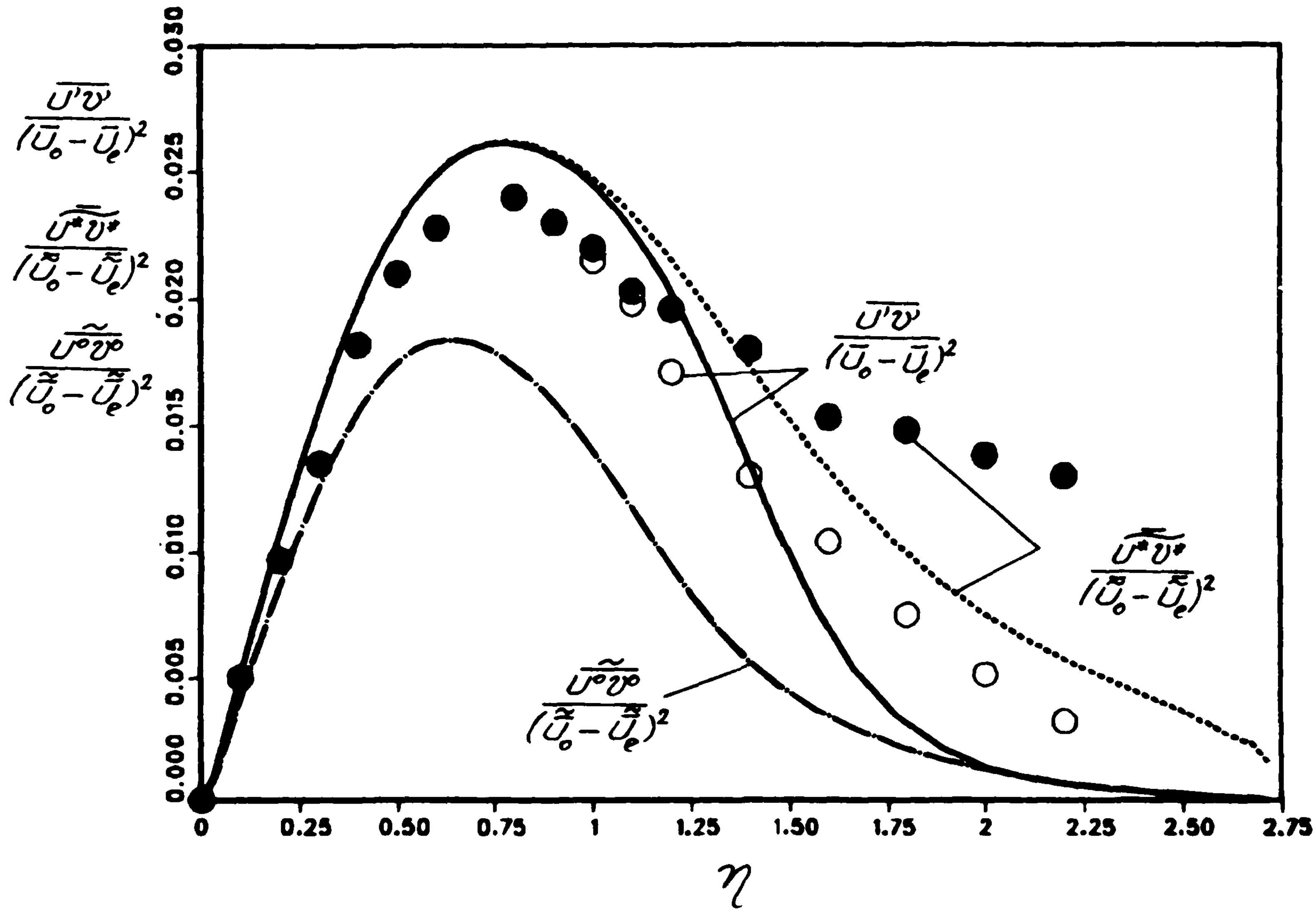
$\frac{\bar{U}-\bar{U}_e}{\bar{U}_o-\bar{U}_e}$
$\frac{\tilde{U}-\tilde{U}_e}{\bar{U}_o-\bar{U}_e}$
$\frac{\tilde{\tilde{U}}-\tilde{\tilde{U}}_e}{\bar{U}_o-\bar{U}_e}$
1.2
1
0.8
0.6
0.4
0.2
0
0.25
0.50
0.75
1
1.25
1.50
1.75
2
2.25
2.50
2.75
η

Fig. 3



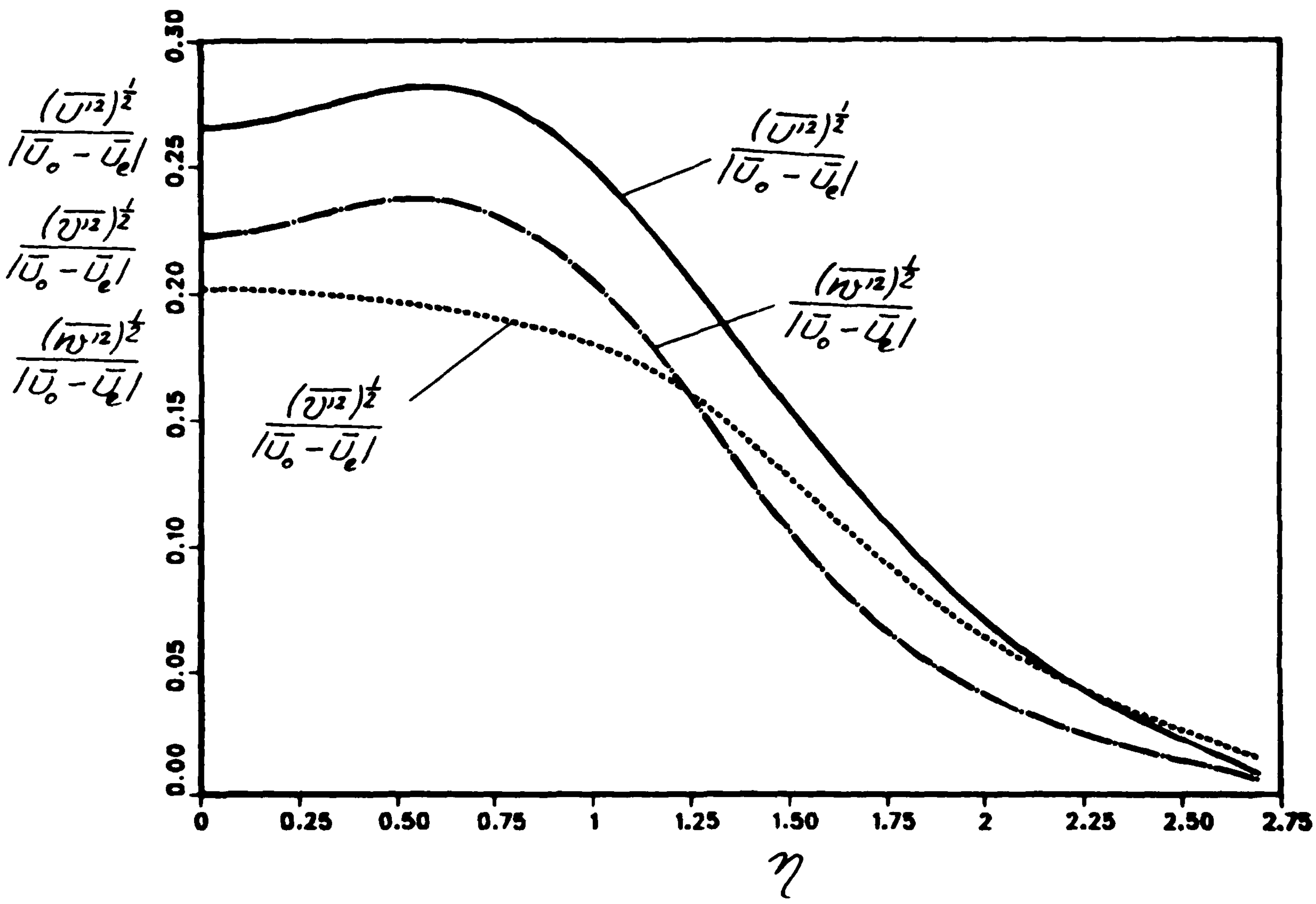
1.2
1
0.8
0.6
0.4
0.2
0
0.25
0.50
0.75
1
1.25
1.50
1.75
2
2.25
2.50
2.75
$\frac{\tilde{\bar{U}} - \tilde{\bar{U}}_e}{\bar{U}_o - \bar{U}_e}$
η

Fig. 4



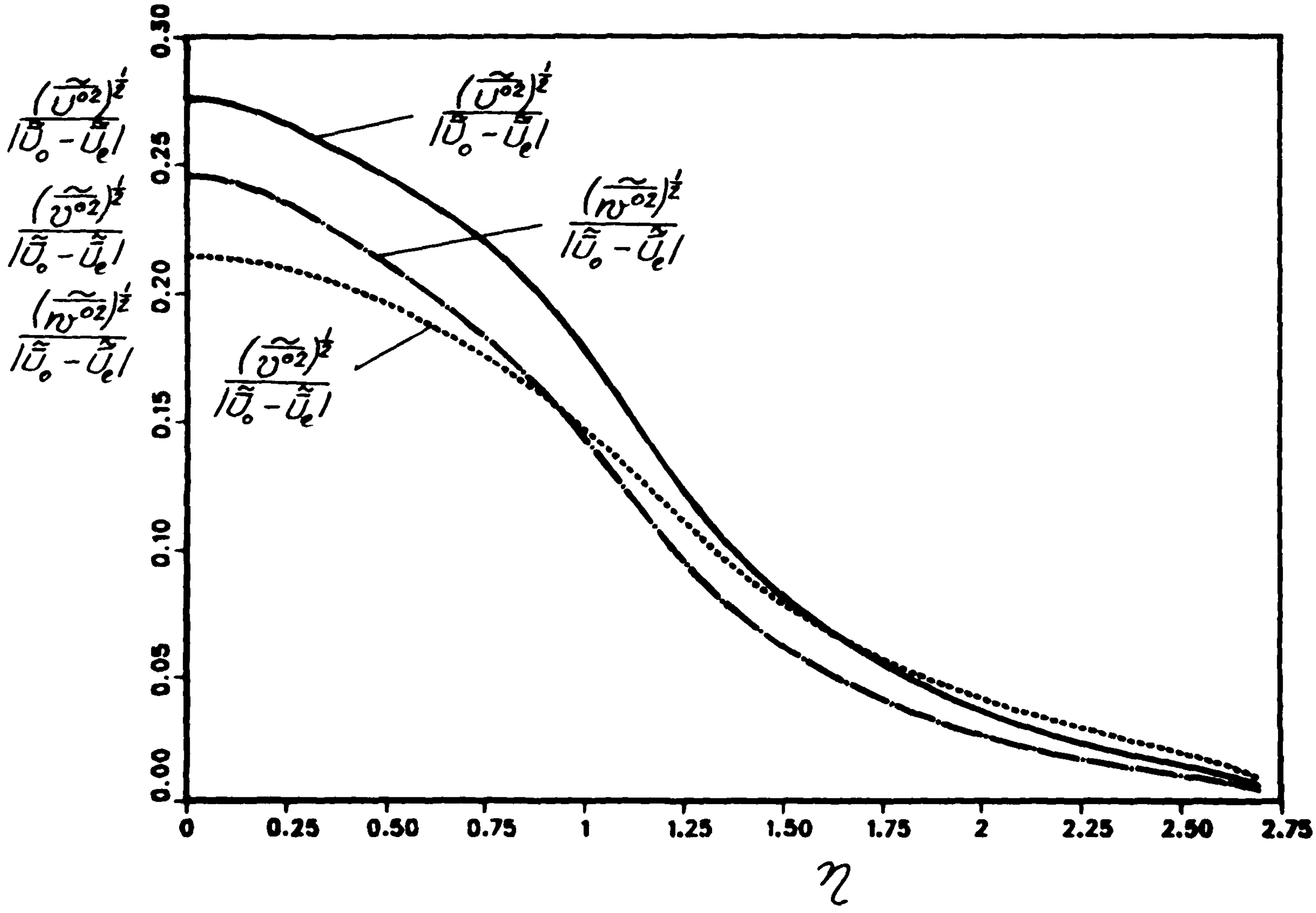
$\frac{\overline{u'v'}}{(\bar{U}_o-\bar{U}_e)^2}$
$\frac{\overline{u^*v^*}}{(\tilde{\bar{U}}_o-\tilde{\bar{U}}_e)^2}$
$\frac{\overline{u^\circ v^\circ}}{(\tilde{\bar{U}}_o-\tilde{\bar{U}}_e)^2}$
0.030
0.025
0.020
0.015
0.010
0.005
0.000
0
0.25
0.50
0.75
1
1.25
1.50
1.75
2
2.25
2.50
2.75
η

Fig. 5



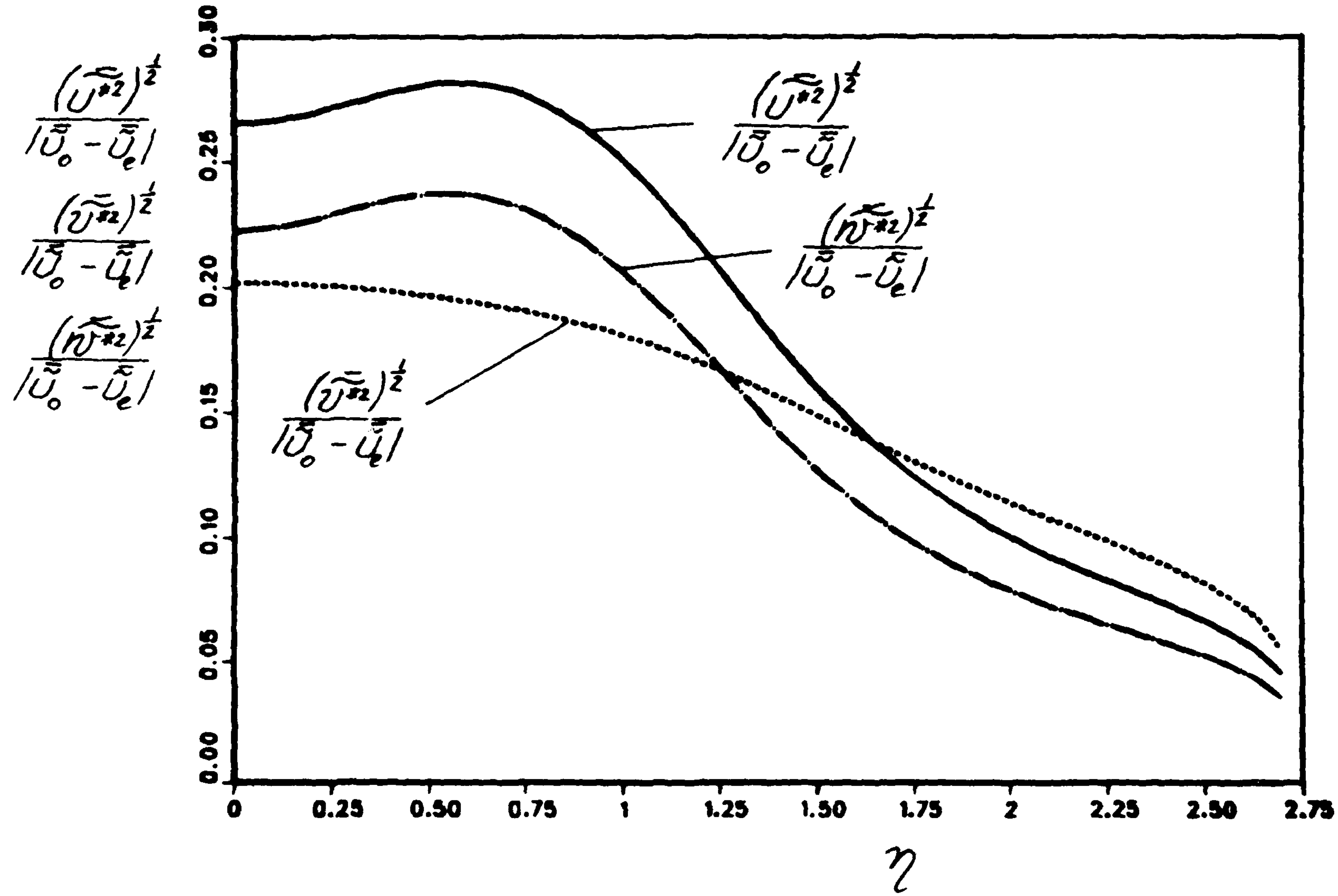
$\frac{(\overline{u'^2})^{\frac{1}{2}}}{|\bar{U}_o - \bar{U}_e|}$
$\frac{(\overline{v'^2})^{\frac{1}{2}}}{|\bar{U}_o - \bar{U}_e|}$
$\frac{(\overline{w'^2})^{\frac{1}{2}}}{|\bar{U}_o - \bar{U}_e|}$
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
0.25
0.50
0.75
1
1.25
1.50
1.75
2
2.25
2.50
2.75
η

Fig. 6



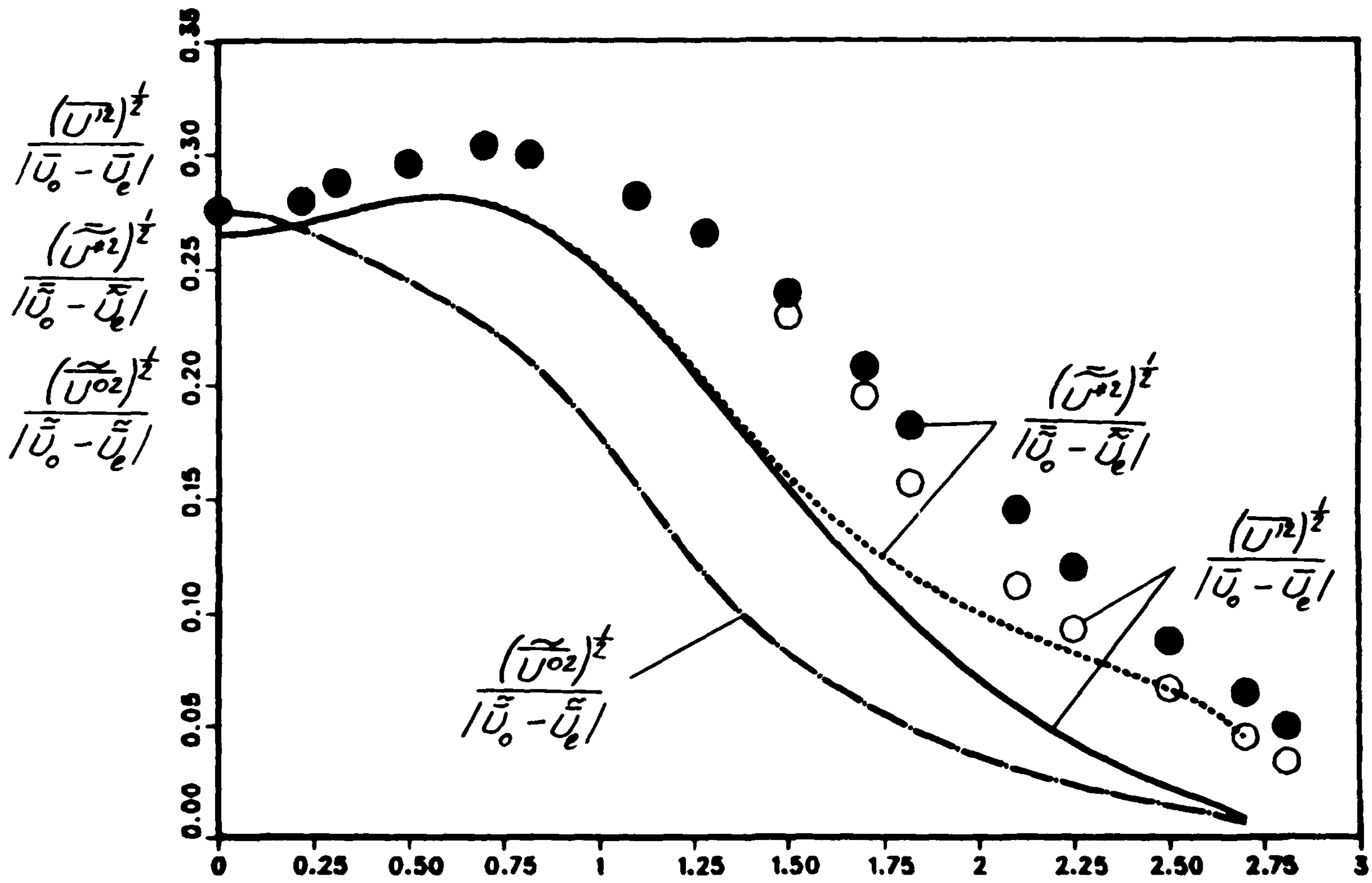
$\frac{(\widetilde{u'^2})^{1/2}}{|\bar{U}_o - \bar{U}_e|}$
$\frac{(\widetilde{v'^2})^{1/2}}{|\bar{U}_o - \bar{U}_e|}$
$\frac{(\widetilde{w'^2})^{1/2}}{|\bar{U}_o - \bar{U}_e|}$
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
0.25
0.50
0.75
1
1.25
1.50
1.75
2
2.25
2.50
2.75
η

Fig. 7



$\frac{(\overline{u^{*2}})^{\frac{1}{2}}}{|\bar{\bar{U}}_o-\bar{\bar{U}}_e|}$
$\frac{(\overline{v^{*2}})^{\frac{1}{2}}}{|\bar{\bar{U}}_o-\bar{\bar{U}}_e|}$
$\frac{(\overline{w^{*2}})^{\frac{1}{2}}}{|\bar{\bar{U}}_o-\bar{\bar{U}}_e|}$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0
0.25
0.50
0.75
1
1.25
1.50
1.75
2
2.25
2.50
2.75
η

Fig. 8



0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
0.25
0.50
0.75
1
1.25
1.50
1.75
2
2.25
2.50
2.75
3

Fig. 9

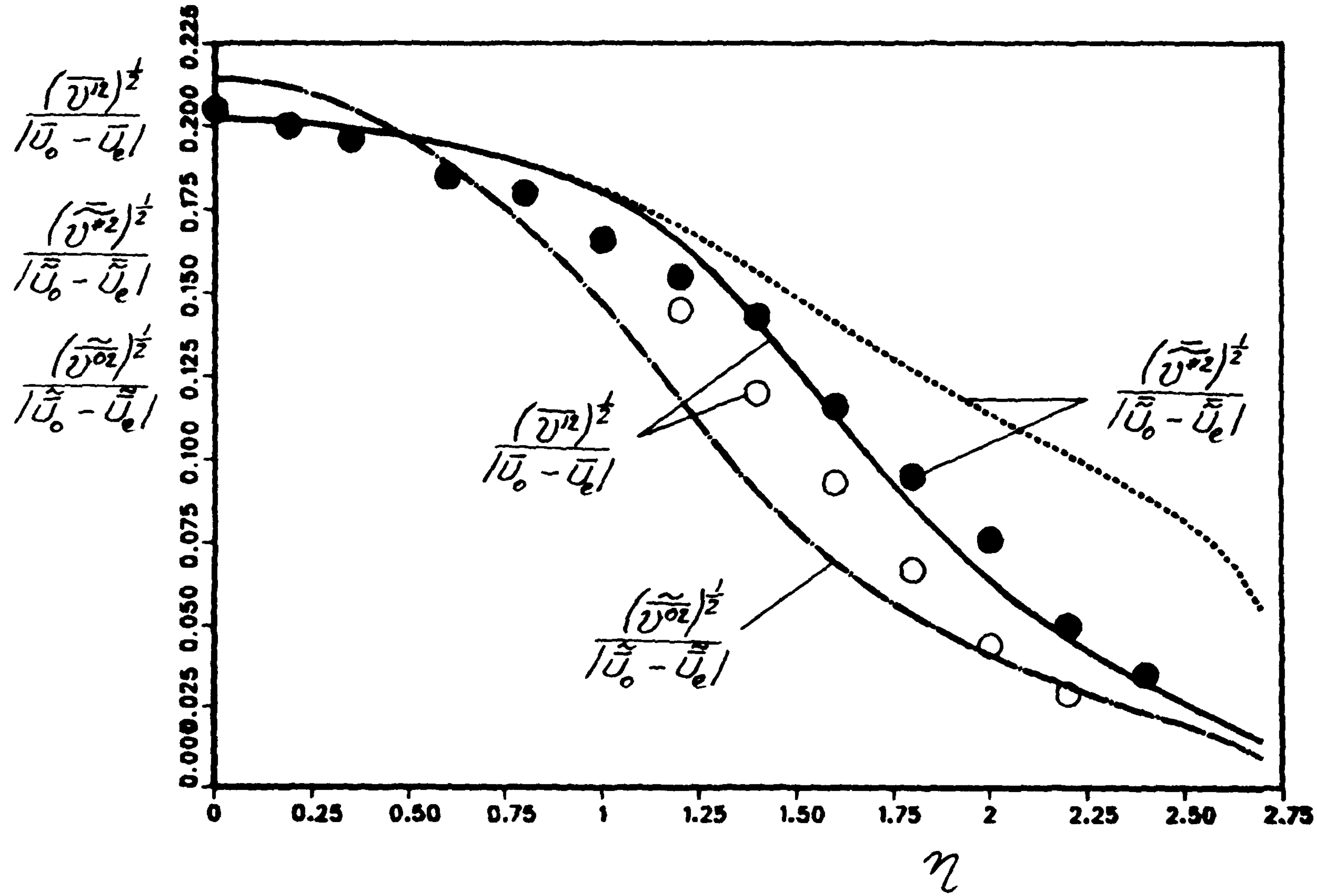
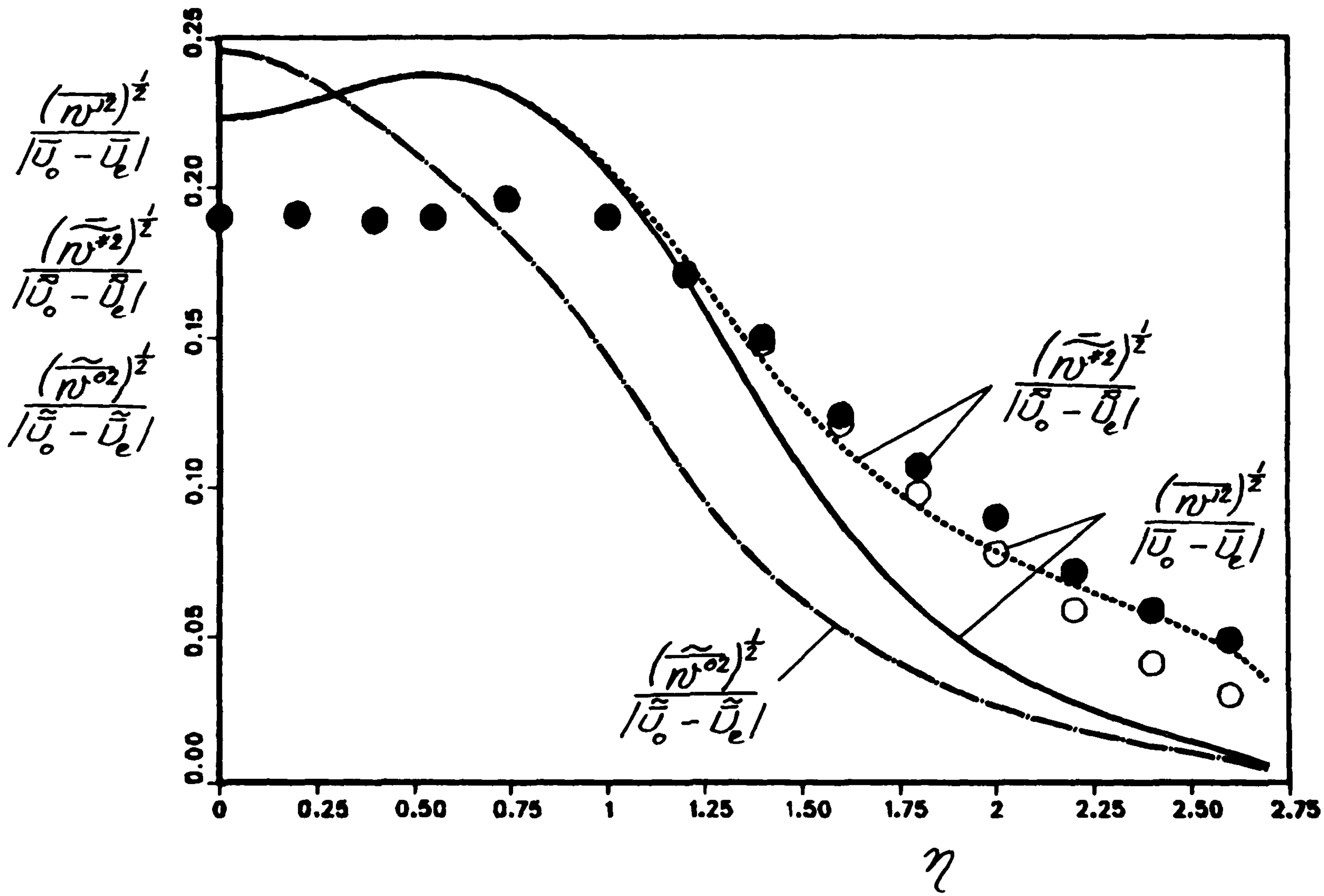

$\frac{(\overline{v'^2})^{\frac{1}{2}}}{|\bar{U}_o - \bar{U}_e|}$
$\frac{(\widetilde{v^{*2}})^{\frac{1}{2}}}{|\bar{\bar{U}}_o - \bar{\bar{U}}_e|}$
$\frac{(\widetilde{v^{o2}})^{\frac{1}{2}}}{|\tilde{\bar{U}}_o - \bar{\bar{U}}_e|}$
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
0.225
0
0.25
0.50
0.75
1
1.25
1.50
1.75
2
2.25
2.50
2.75
η

Fig. 10



$\frac{(\overline{v'^2})^{1/2}}{|\overline{U}_o - \overline{U}_e|}$
$\frac{(\overline{\tilde{v}^{*2}})^{1/2}}{|\bar{\bar{U}}_o - \bar{\bar{U}}_e|}$
$\frac{(\overline{\tilde{v}^{o2}})^{1/2}}{|\tilde{\tilde{U}}_o - \tilde{\tilde{U}}_e|}$
0.00
0.05
0.10
0.15
0.20
0.25
0
0.25
0.50
0.75
1
1.25
1.50
1.75
2
2.25
2.50
2.75
η

Fig. 11

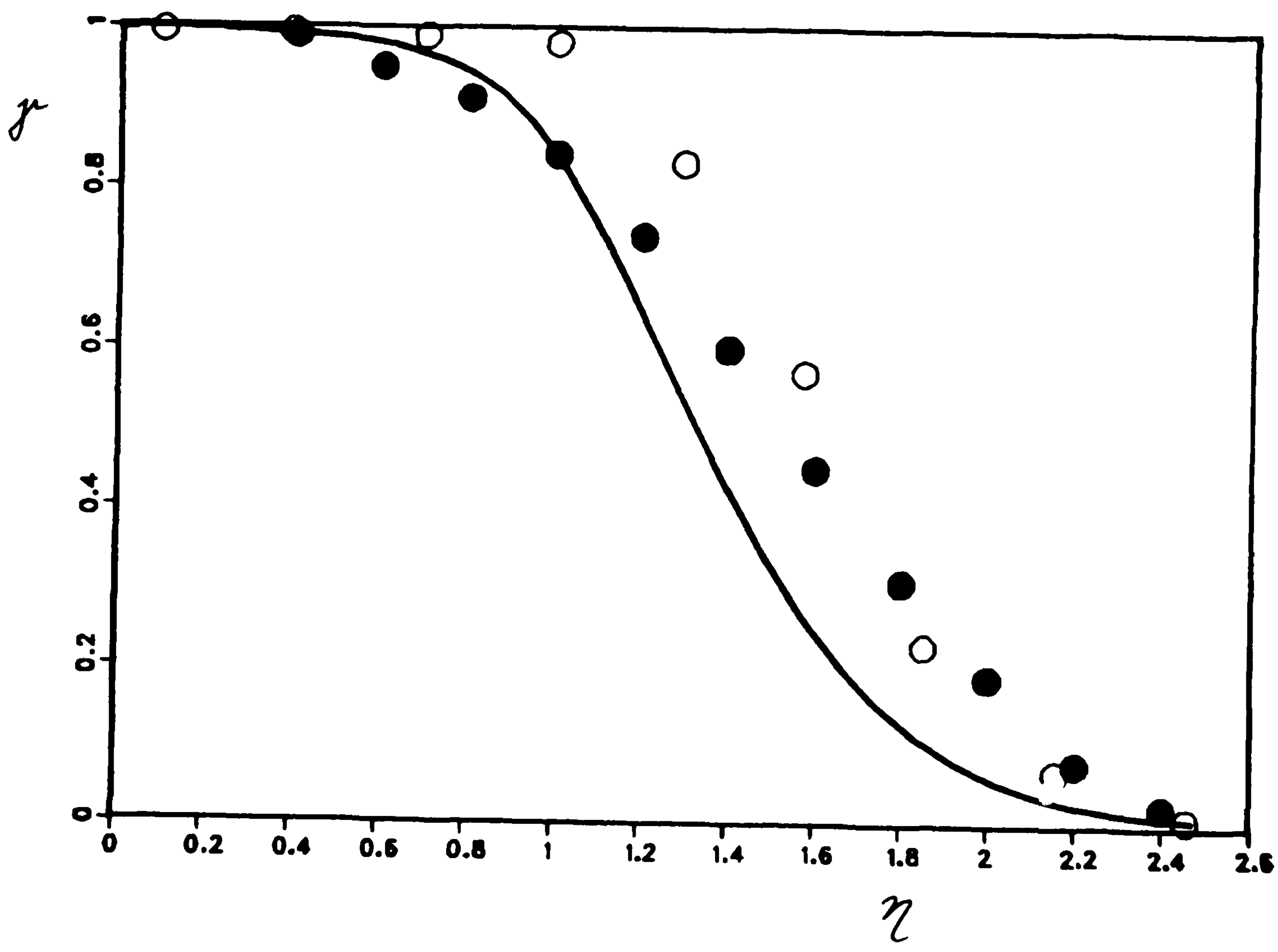
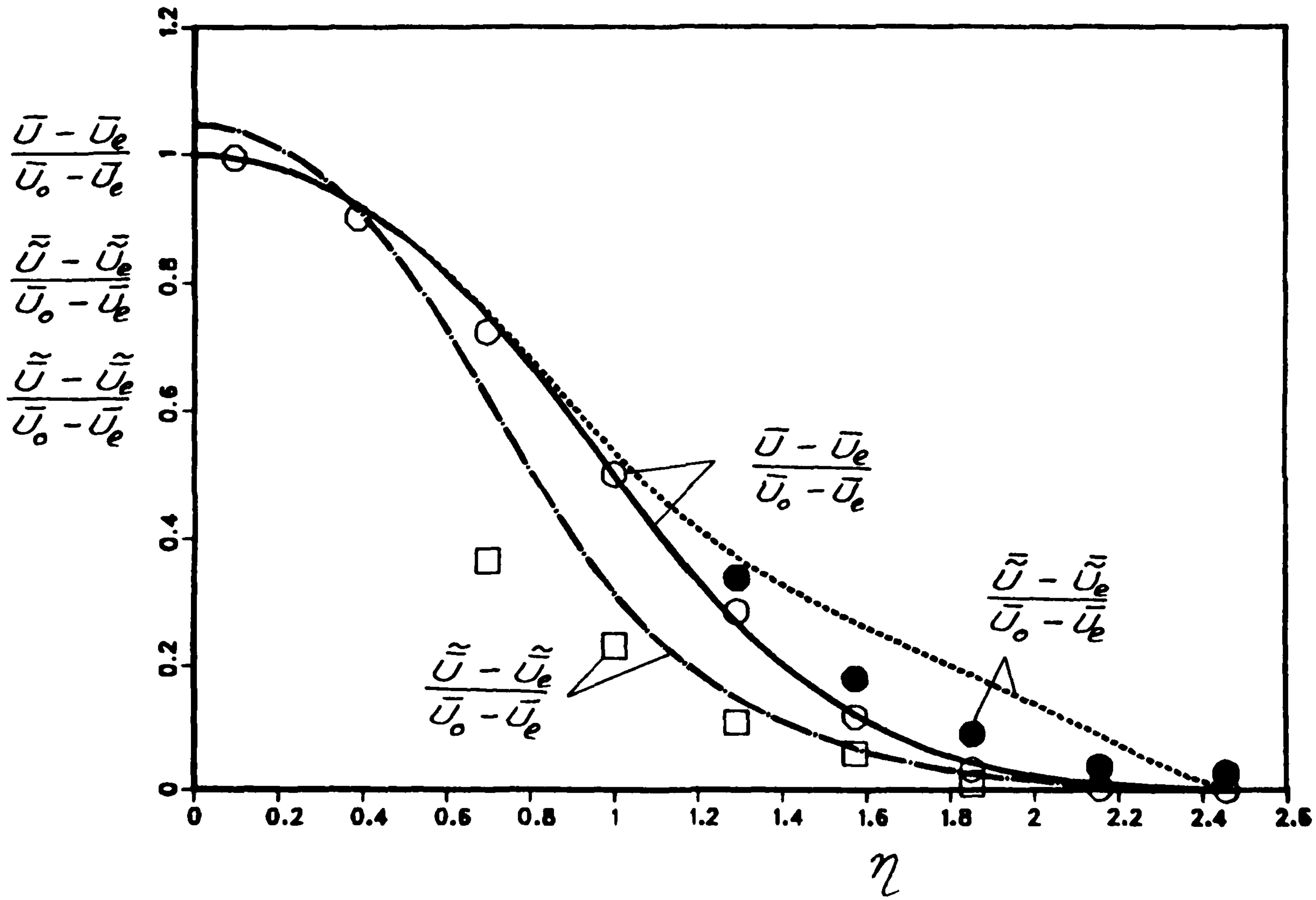
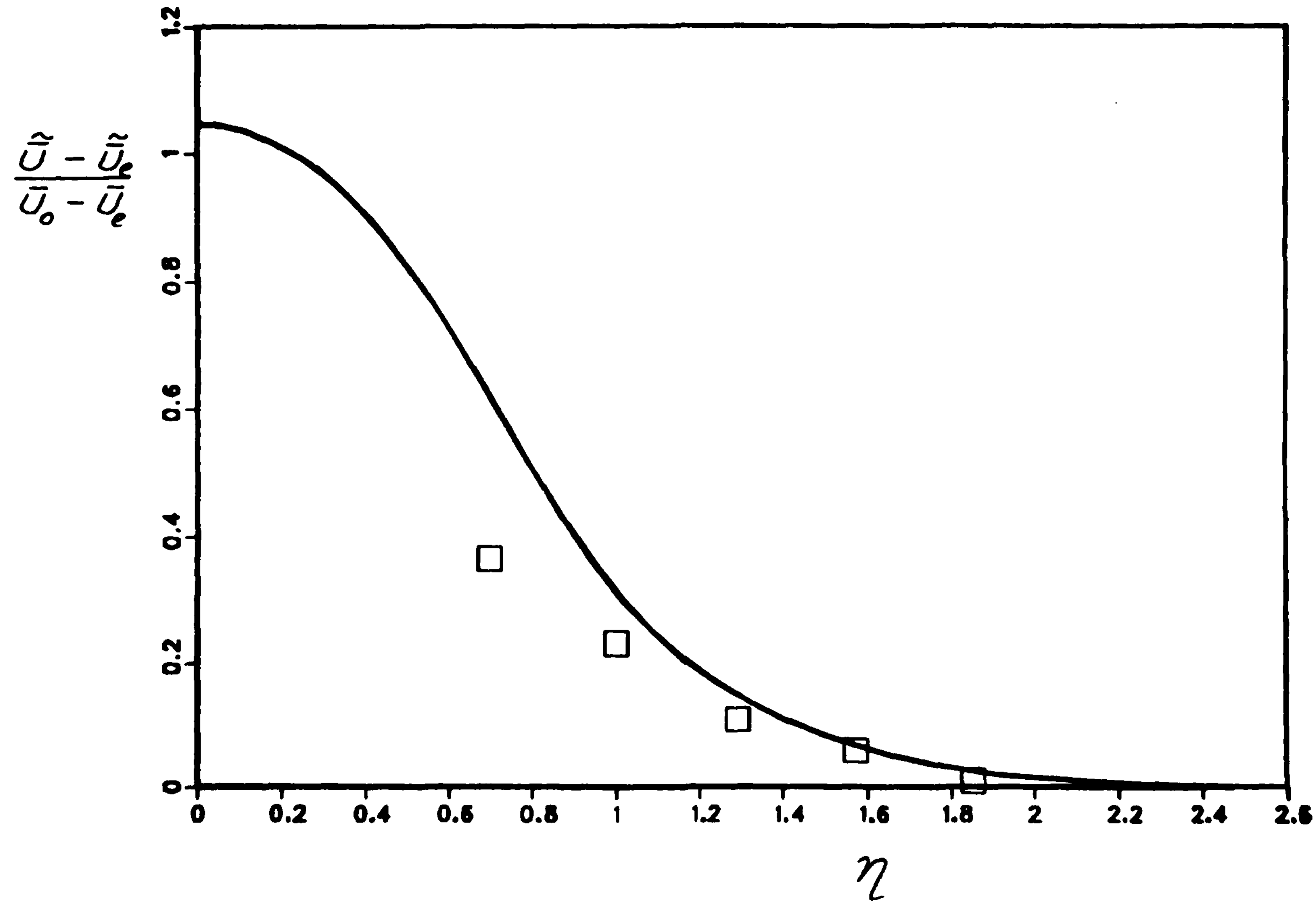


Fig. 12



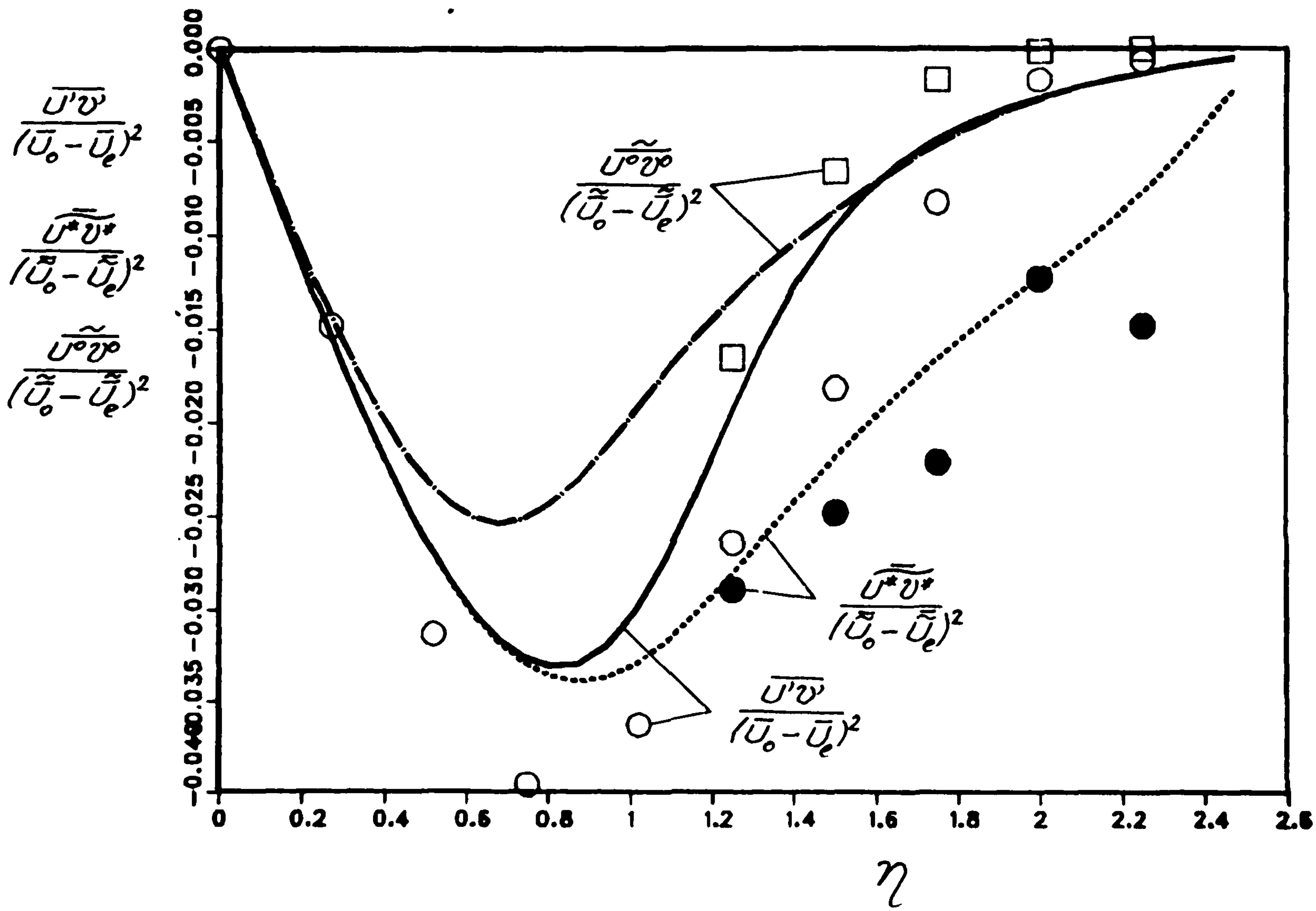
$\frac{\bar{U} - \bar{U}_e}{\bar{U}_o - \bar{U}_e}$
$\frac{\tilde{\bar{U}} - \tilde{\bar{U}}_e}{\bar{U}_o - \bar{U}_e}$
$\frac{\tilde{\tilde{U}} - \tilde{\tilde{U}}_e}{\bar{U}_o - \bar{U}_e}$
1.2
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
η

Fig. 13



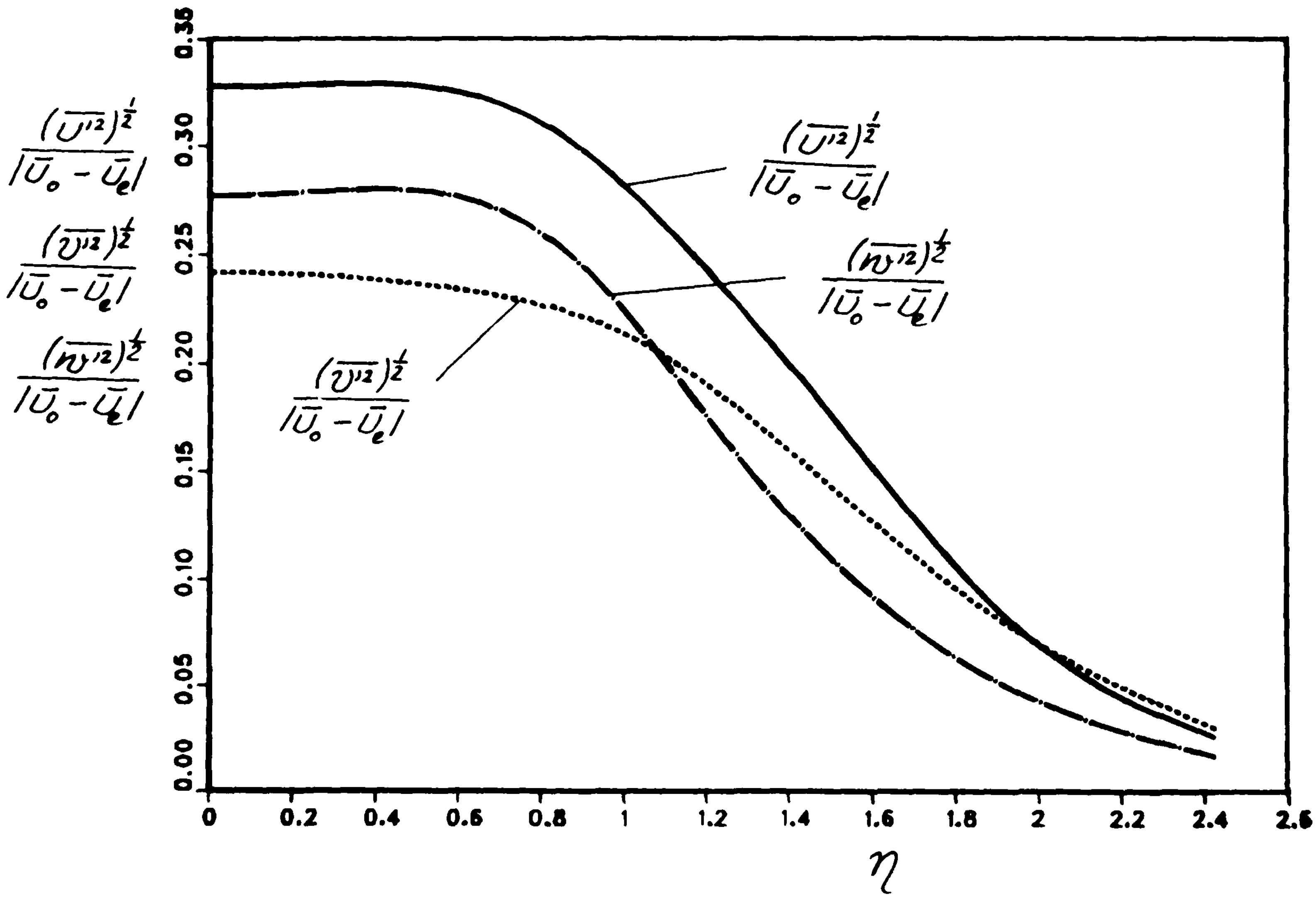
$\frac{\tilde{U} - \tilde{U}_e}{\bar{U}_o - \bar{U}_e}$
1.2
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
η

Fig. 14



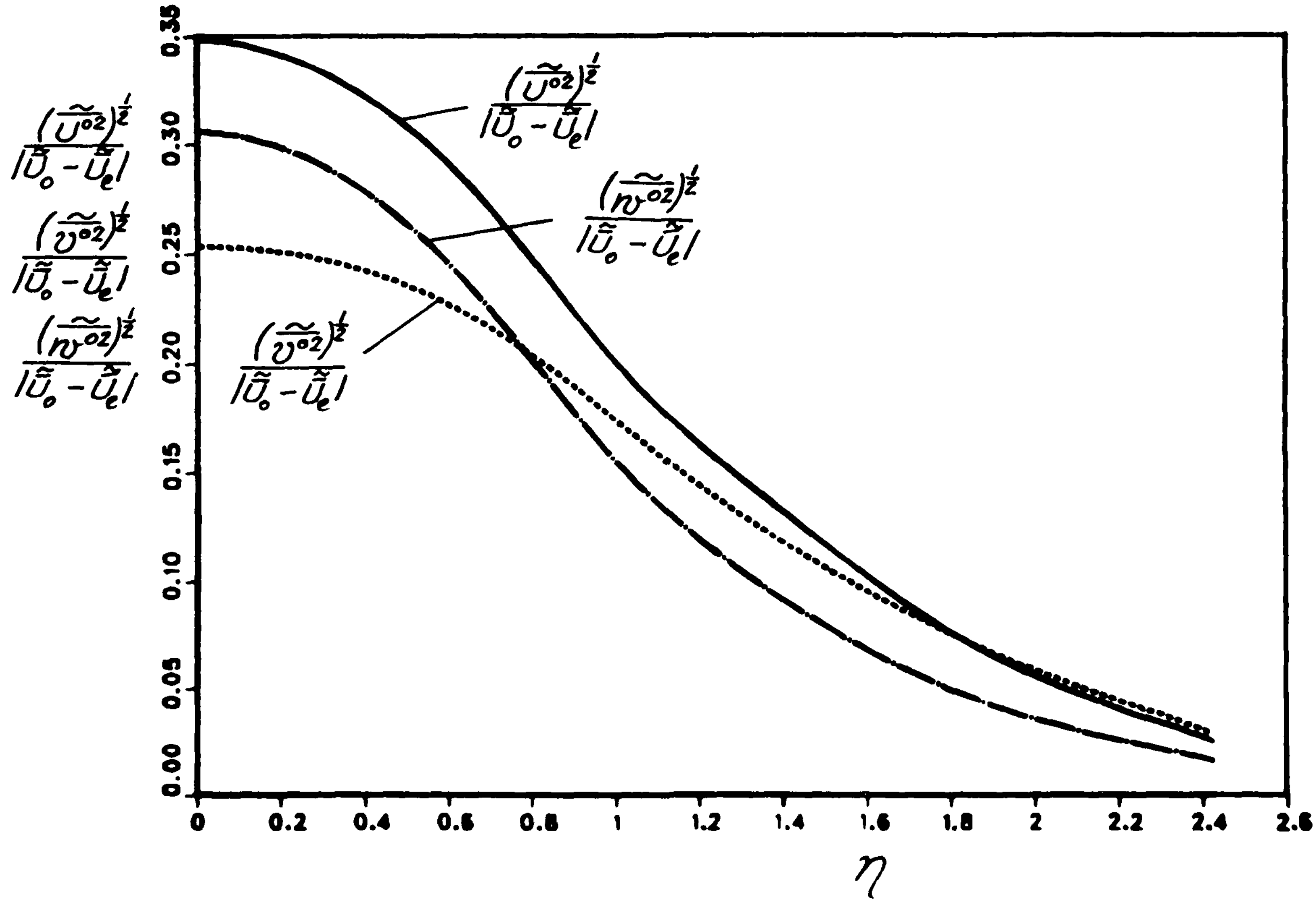
$\frac{\overline{u'v'}}{(\overline{U}_o-\overline{U}_e)^2}$
$\frac{\overline{u^*v^*}}{(\bar{\bar{U}}_o-\bar{\bar{U}}_e)^2}$
$\frac{\widetilde{u^\circ v^\circ}}{(\tilde{\bar{U}}_o-\tilde{\bar{U}}_e)^2}$
0.000
-0.005
-0.010
-0.015
-0.020
-0.025
-0.030
-0.035
-0.040
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
η

Fig. 15



$\frac{(\overline{u'^2})^{\frac{1}{2}}}{|\bar{U}_o - \bar{U}_e|}$
$\frac{(\overline{v'^2})^{\frac{1}{2}}}{|\bar{U}_o - \bar{U}_e|}$
$\frac{(\overline{w'^2})^{\frac{1}{2}}}{|\bar{U}_o - \bar{U}_e|}$
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
η

Fig. 16



$\frac{(\widetilde{u'^2})^{1/2}}{|\tilde{\bar{U}}_o - \tilde{\bar{U}}_e|}$
$\frac{(\widetilde{v'^2})^{1/2}}{|\tilde{\bar{U}}_o - \tilde{\bar{U}}_e|}$
$\frac{(\widetilde{w'^2})^{1/2}}{|\tilde{\bar{U}}_o - \tilde{\bar{U}}_e|}$
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
η

Fig. 17

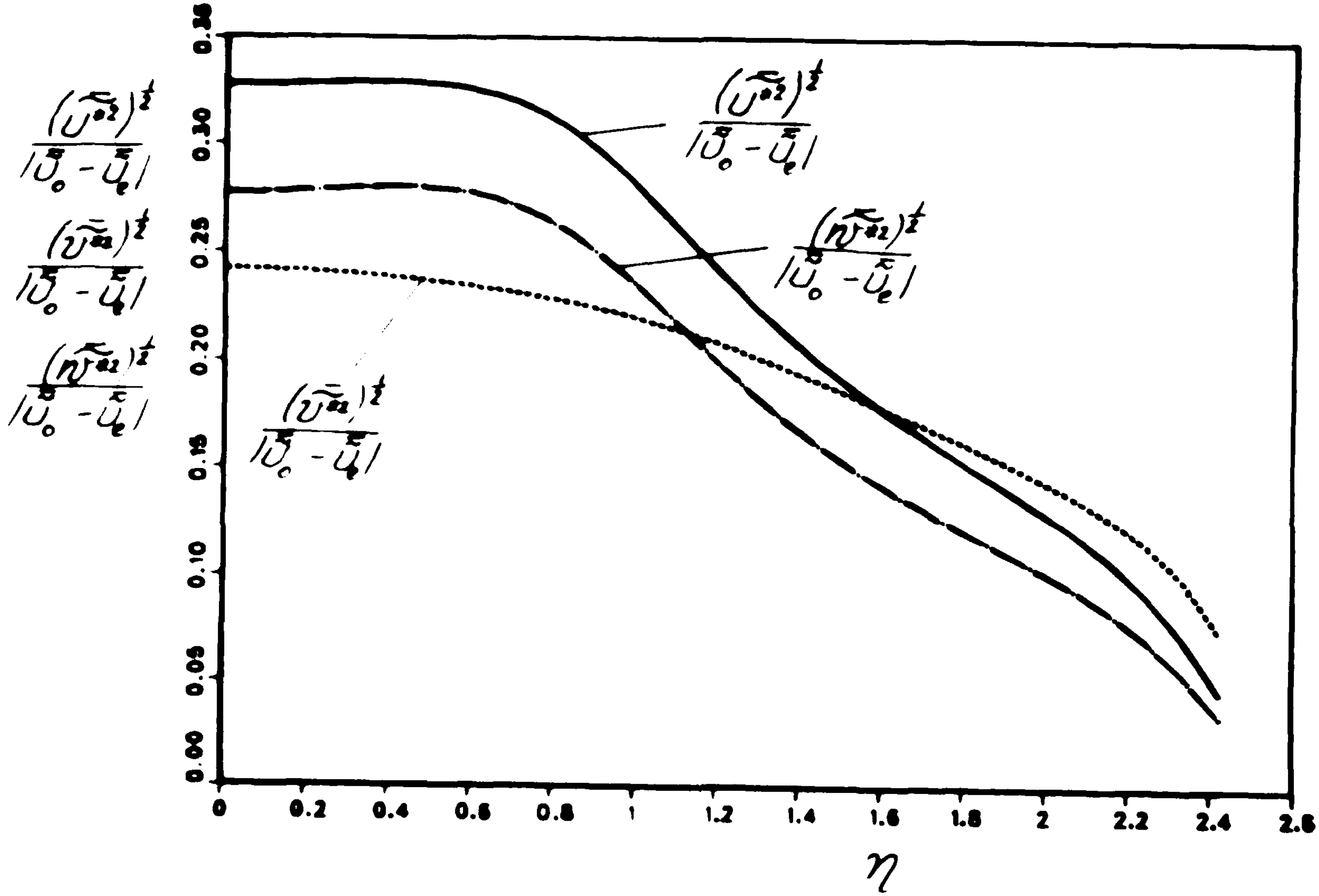
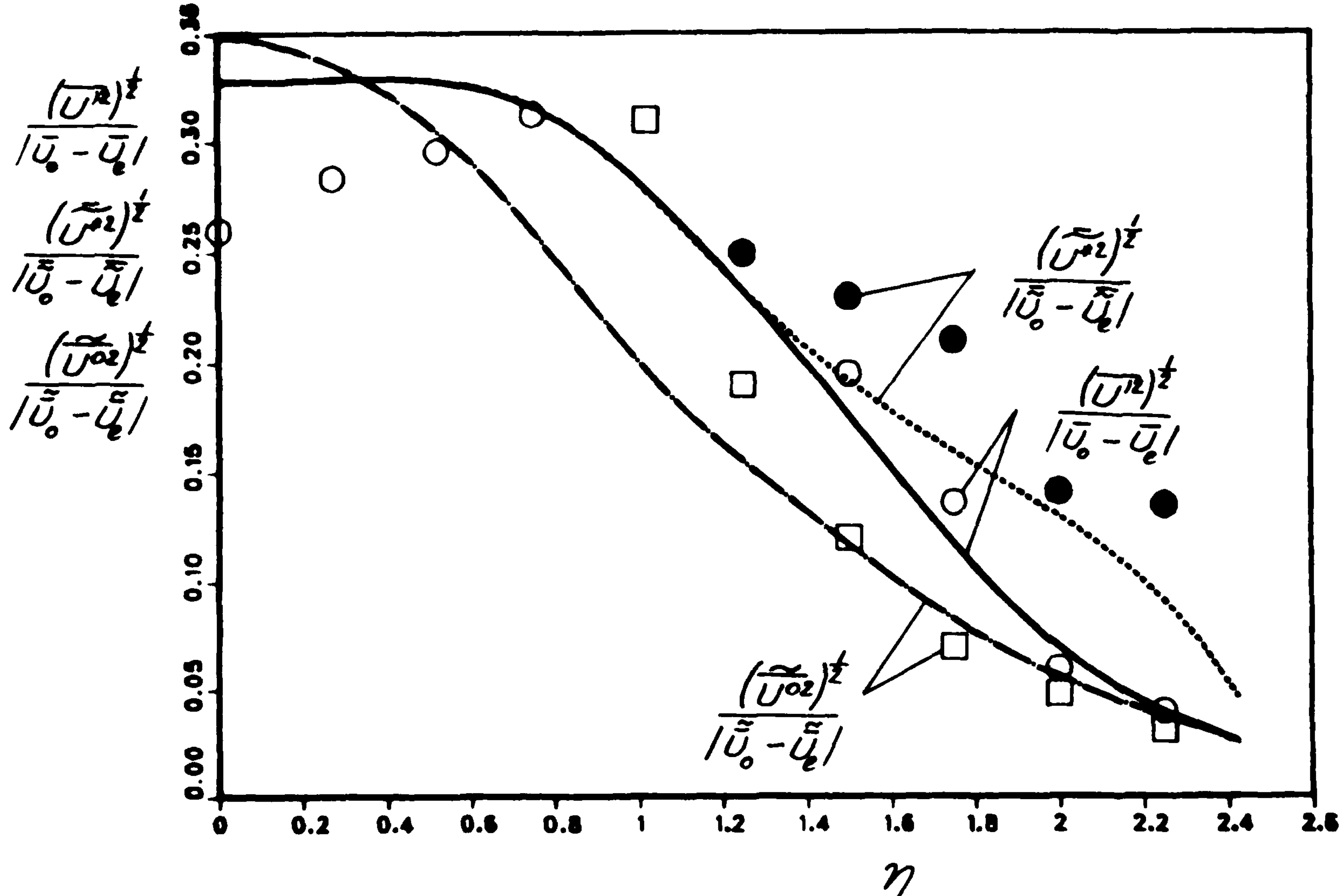
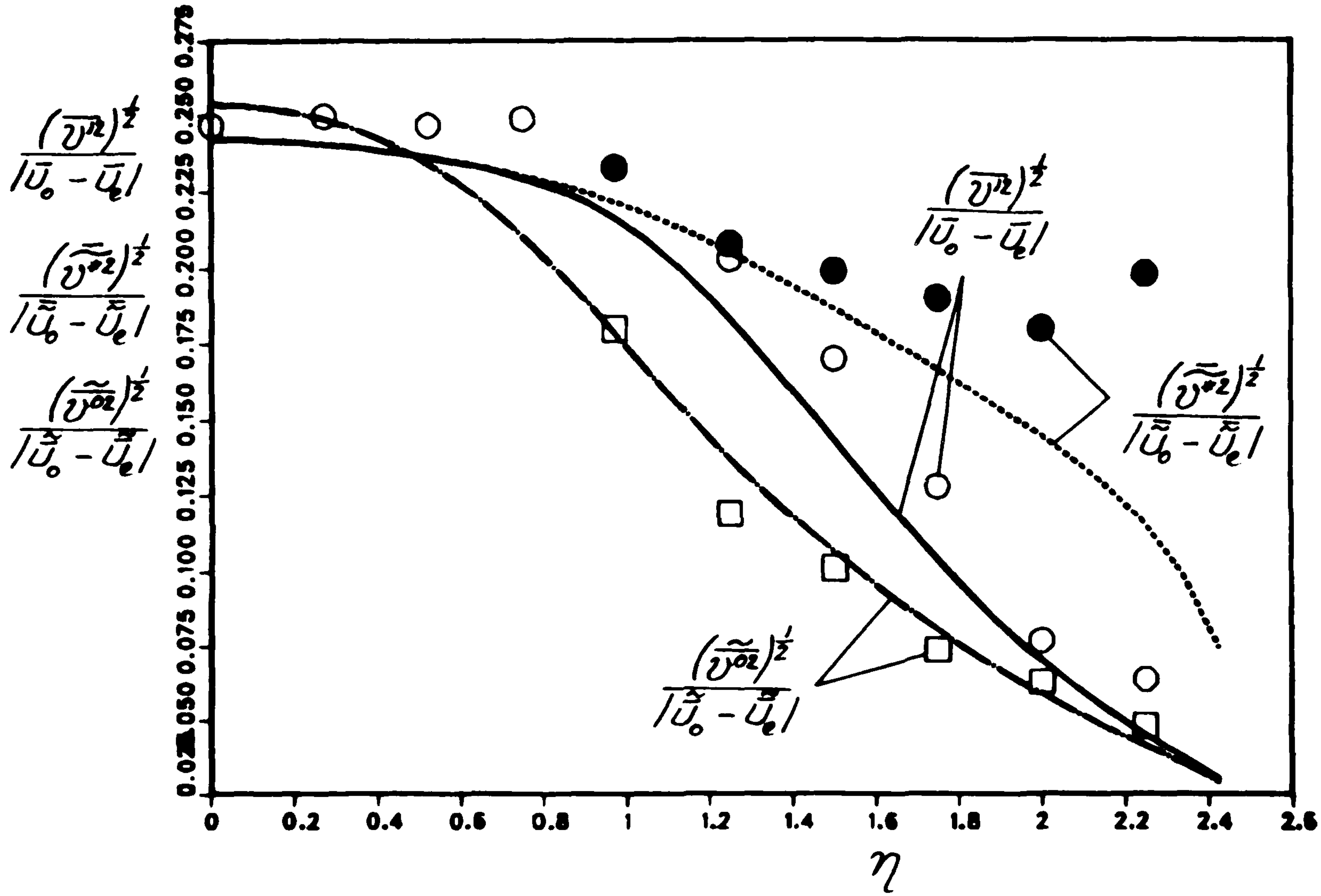

0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
η

Fig. 18



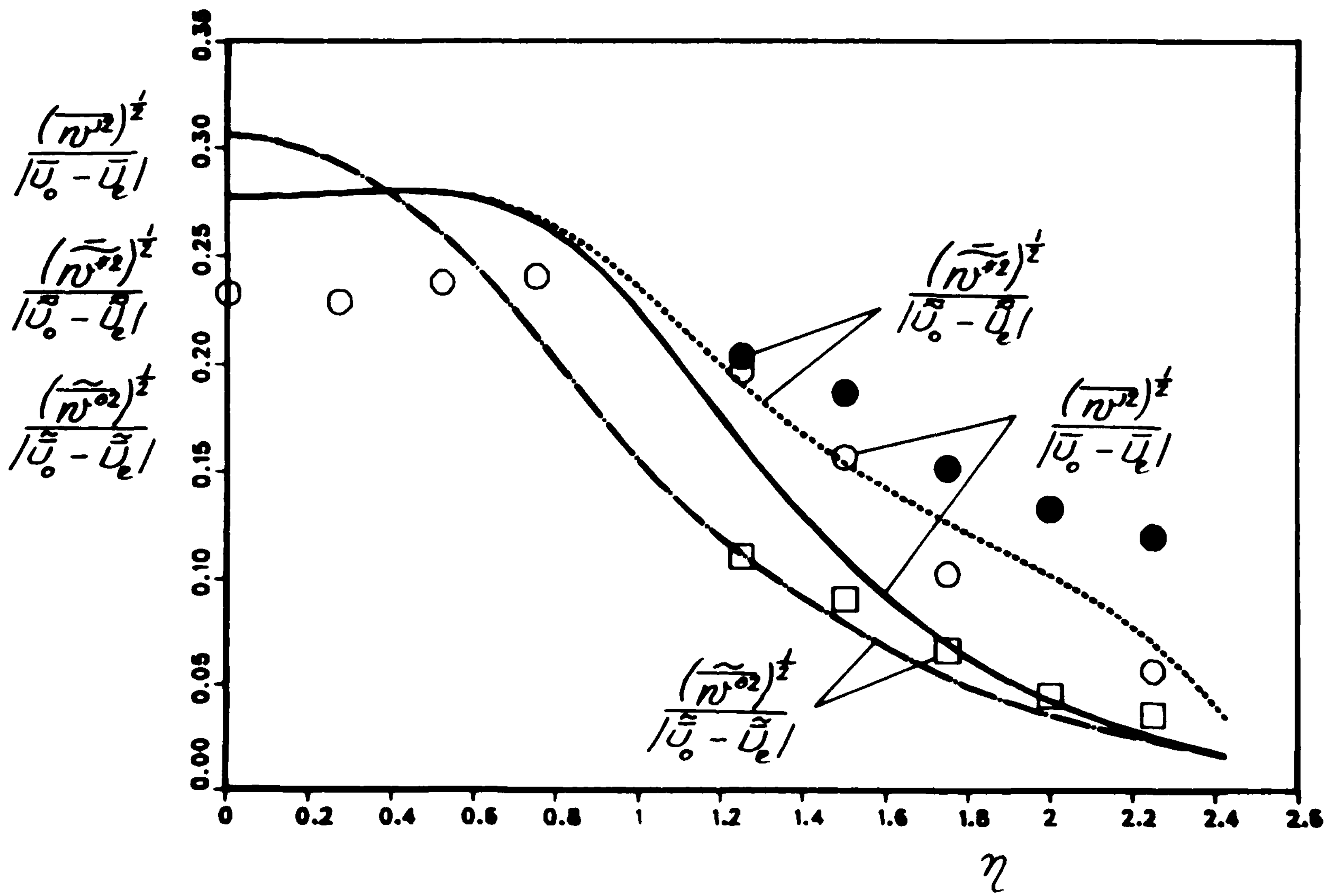
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
η

Fig. 19



$\frac{(\overline{v'^2})^{1/2}}{|\bar{U}_o - \bar{U}_e|}$
$\frac{(\overline{\tilde{v}^{*2}})^{1/2}}{|\bar{\bar{U}}_o - \bar{\bar{U}}_e|}$
$\frac{(\overline{\tilde{v}^{o2}})^{1/2}}{|\hat{\bar{U}}_o - \bar{\bar{U}}_e|}$
0.025 0.050 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2 2.2 2.4 2.6
η

Fig. 20



$\frac{(\overline{v'^2})^{\frac{1}{2}}}{|\overline{U}_o - \overline{U}_e|}$
$\frac{(\overline{\tilde{v}^{*2}})^{\frac{1}{2}}}{|\bar{\bar{U}}_o - \bar{\bar{U}}_e|}$
$\frac{(\overline{\tilde{v}^{o2}})^{\frac{1}{2}}}{|\tilde{\bar{U}}_o - \tilde{\bar{U}}_e|}$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
η

Fig. 21

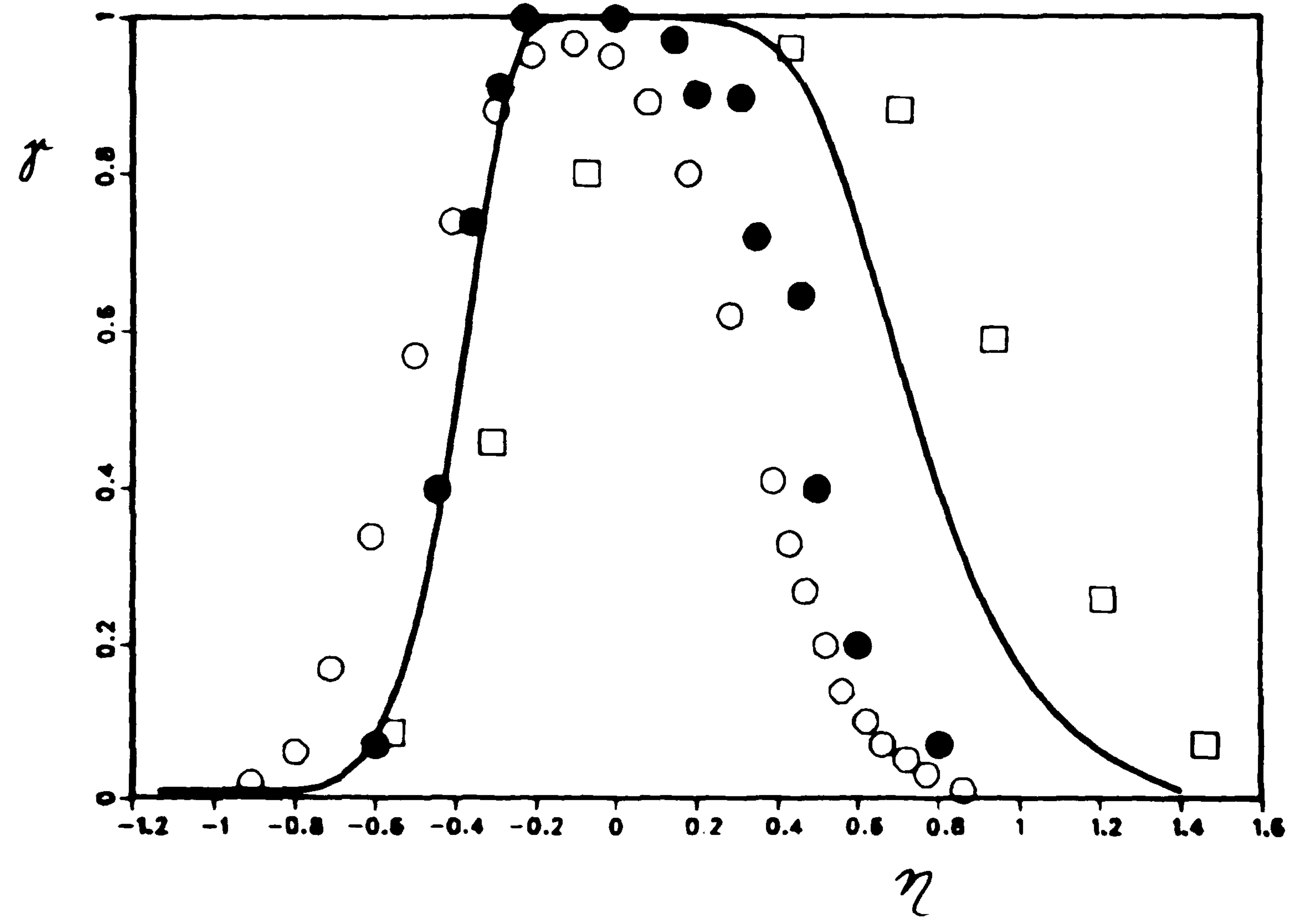
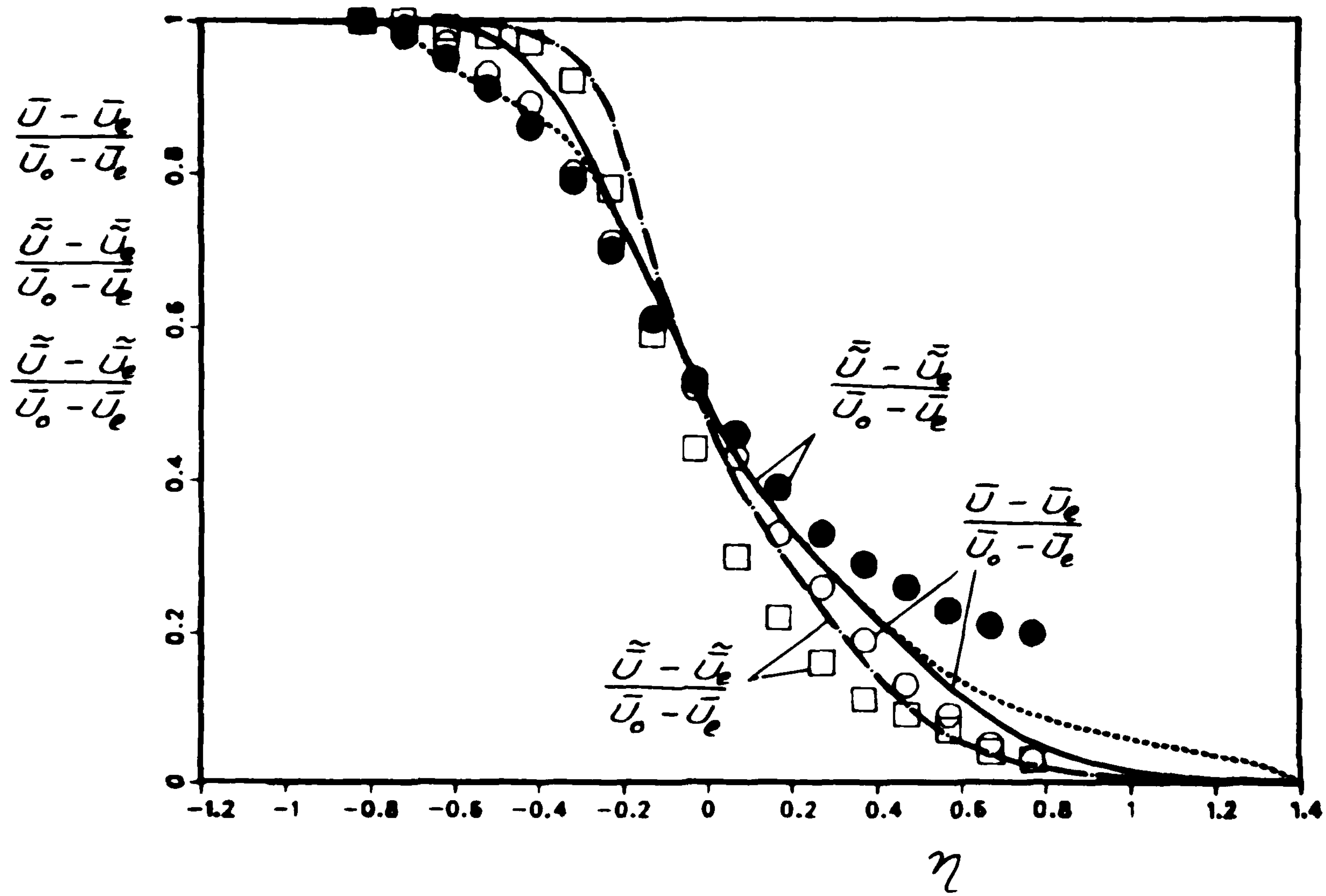
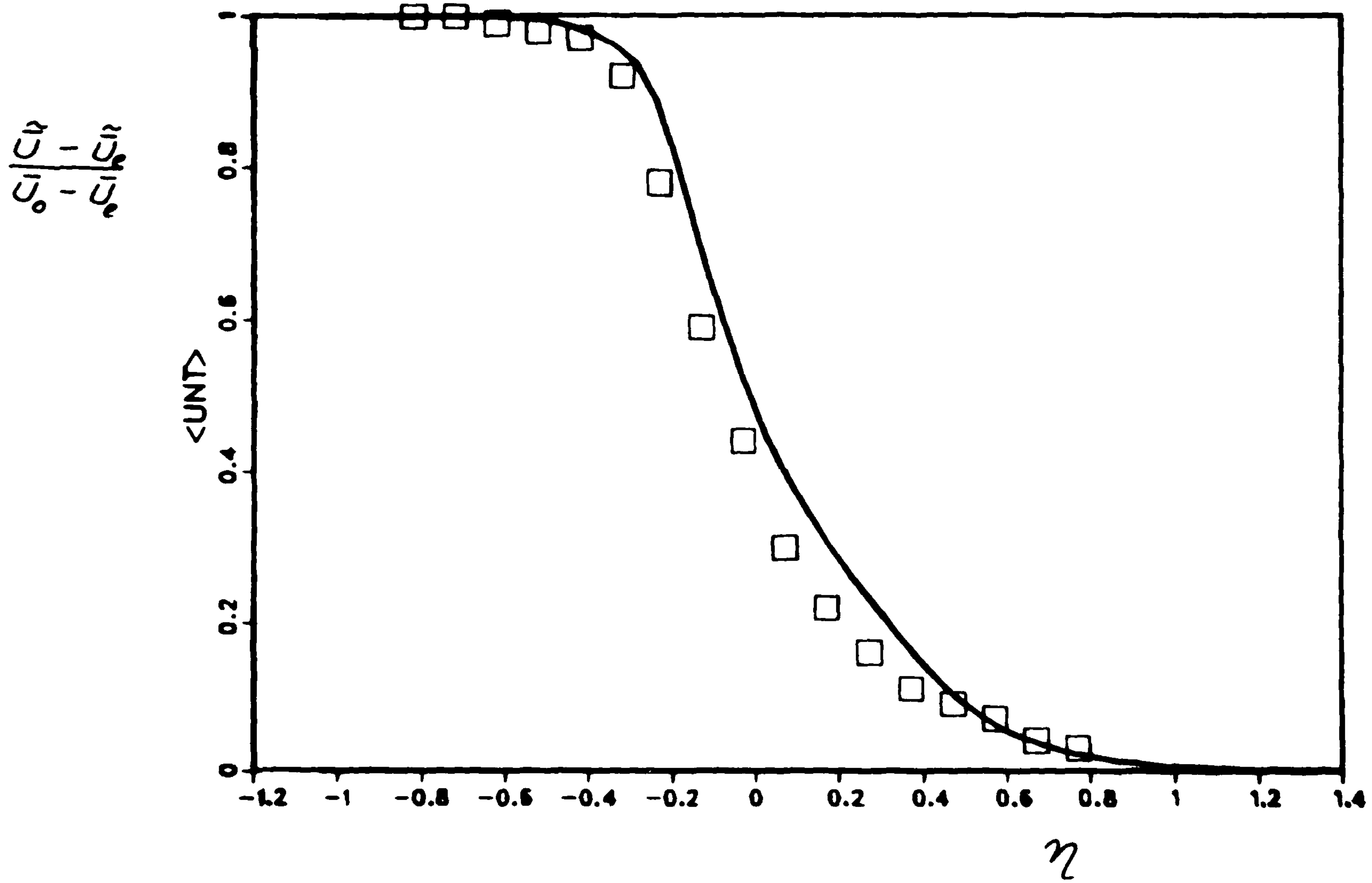


Fig. 22



$\frac{\bar{U}-\bar{U}_e}{\bar{U}_o-\bar{U}_e}$
$\frac{\tilde{\bar{U}}-\tilde{\bar{U}}_e}{\bar{U}_o-\bar{U}_e}$
$\frac{\tilde{U}-\tilde{U}_e}{\bar{U}_o-\bar{U}_e}$
0
0.2
0.4
0.6
0.8
1
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
η

Fig. 23



$\frac{\tilde{U} - \tilde{U}_e}{\bar{U}_o - \bar{U}_e}$
<UNT>
0
0.2
0.4
0.6
0.8
1
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
η

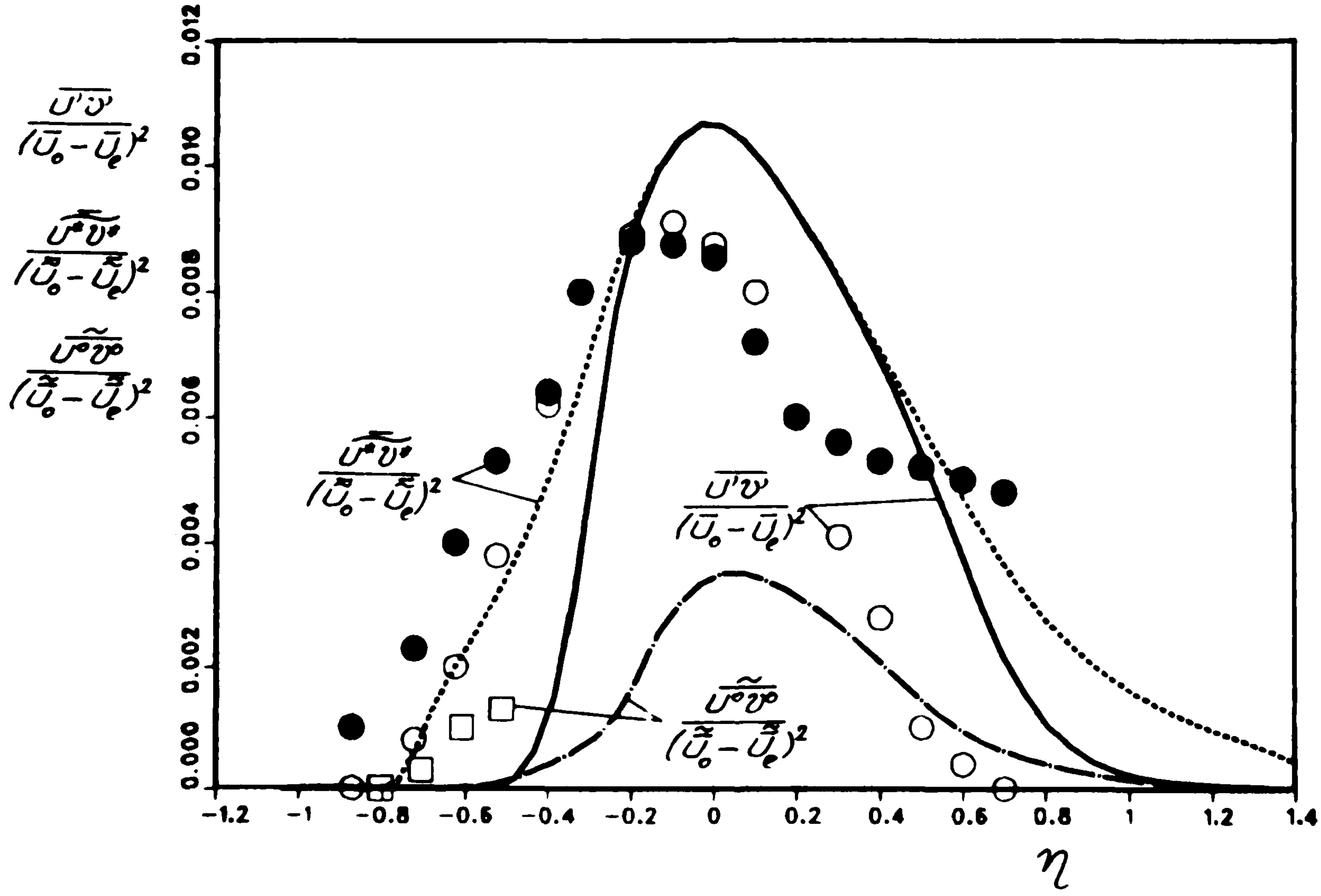

$\frac{\overline{u'v'}}{(\bar{U}_o-\bar{U}_e)^2}$
$\frac{\overline{u^*v^*}}{(\bar{\bar{U}}_o-\bar{\bar{U}}_e)^2}$
$\frac{\overline{u^\circ v^\circ}}{(\tilde{U}_o-\tilde{U}_e)^2}$
0.012
0.010
0.008
0.006
0.004
0.002
0.000
−1.2
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
η

Fig. 24

Fig. 25 .

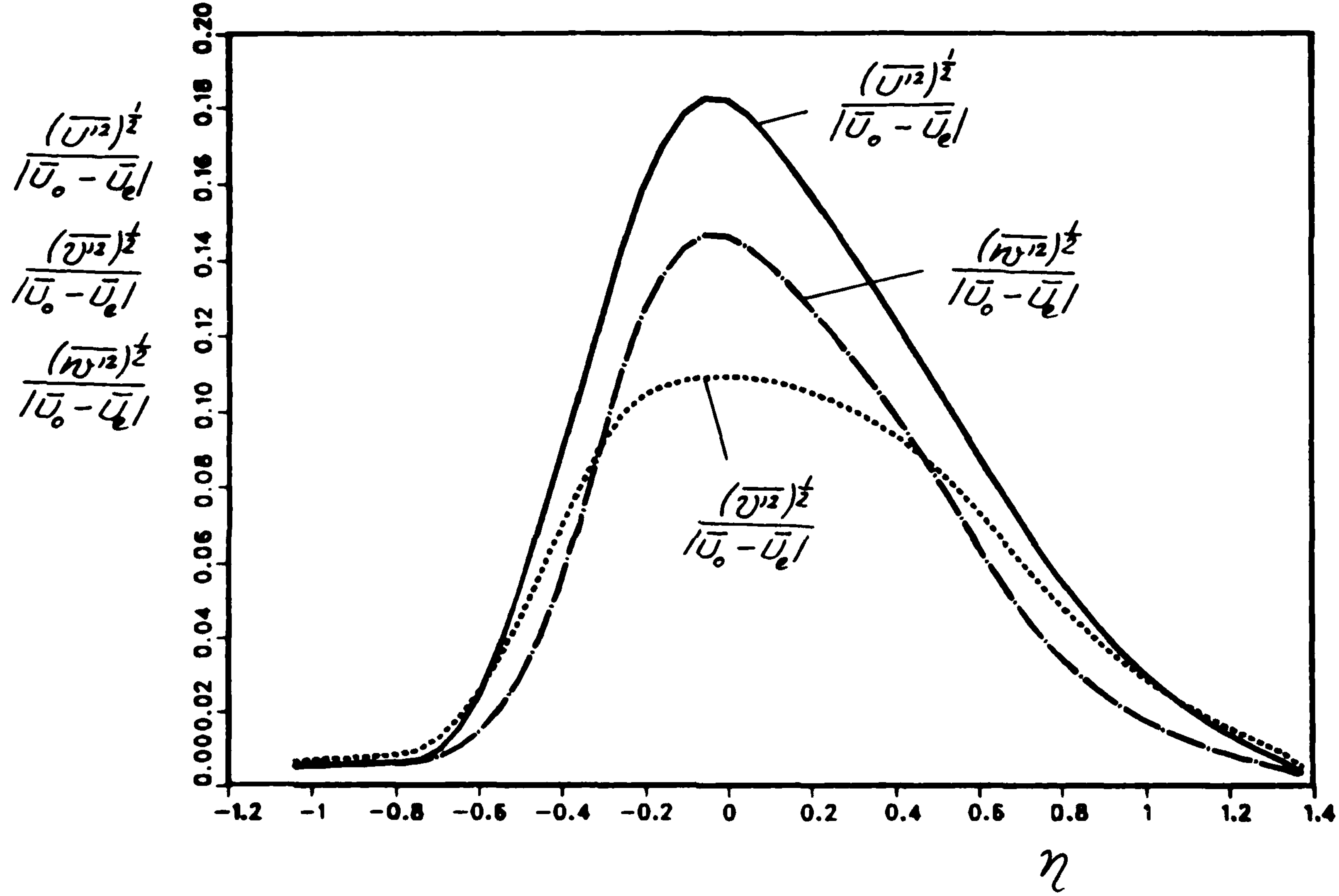
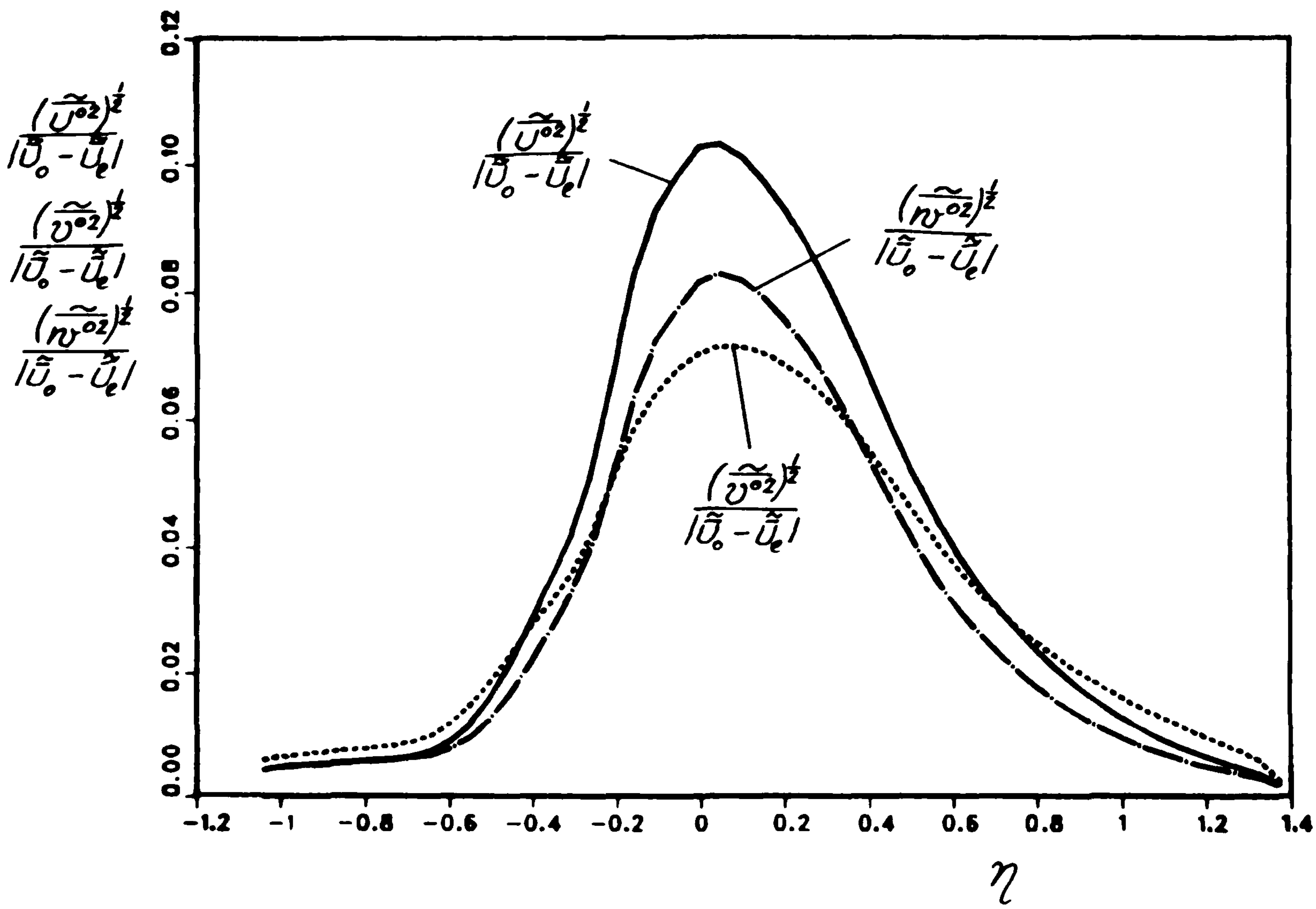
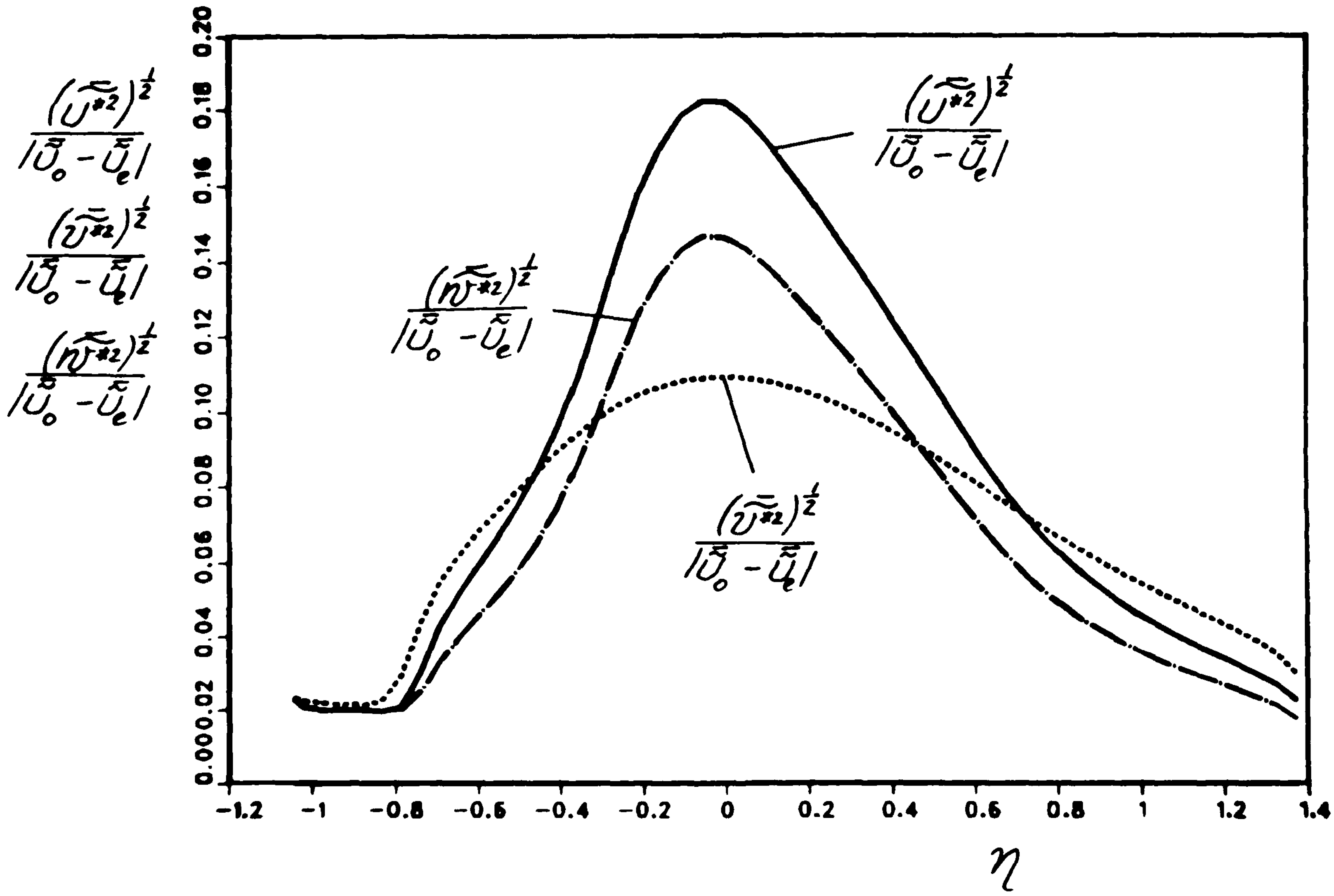

$\frac{(\overline{u'^2})^{1/2}}{|\bar{U}_o - \bar{U}_e|}$
$\frac{(\overline{v'^2})^{1/2}}{|\bar{U}_o - \bar{U}_e|}$
$\frac{(\overline{w'^2})^{1/2}}{|\bar{U}_o - \bar{U}_e|}$
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0.00
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
η

Fig. 26



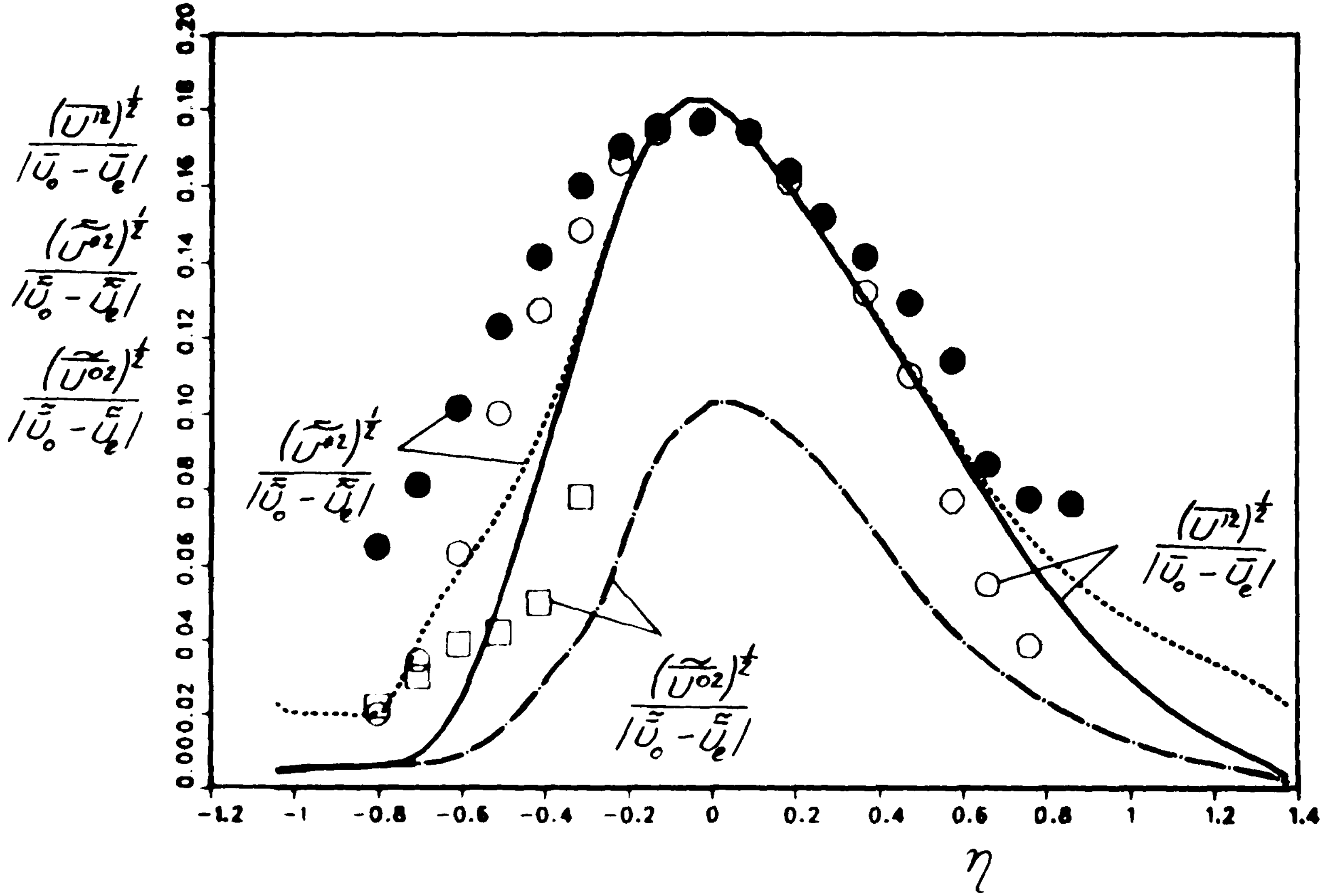
$\frac{(\widetilde{u'^2})^{1/2}}{|\bar{U}_o - \bar{U}_e|}$
$\frac{(\widetilde{v'^2})^{1/2}}{|\bar{U}_o - \bar{U}_e|}$
$\frac{(\widetilde{w'^2})^{1/2}}{|\bar{U}_o - \bar{U}_e|}$
0.12
0.10
0.08
0.06
0.04
0.02
0.00
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
η

Fig. 27



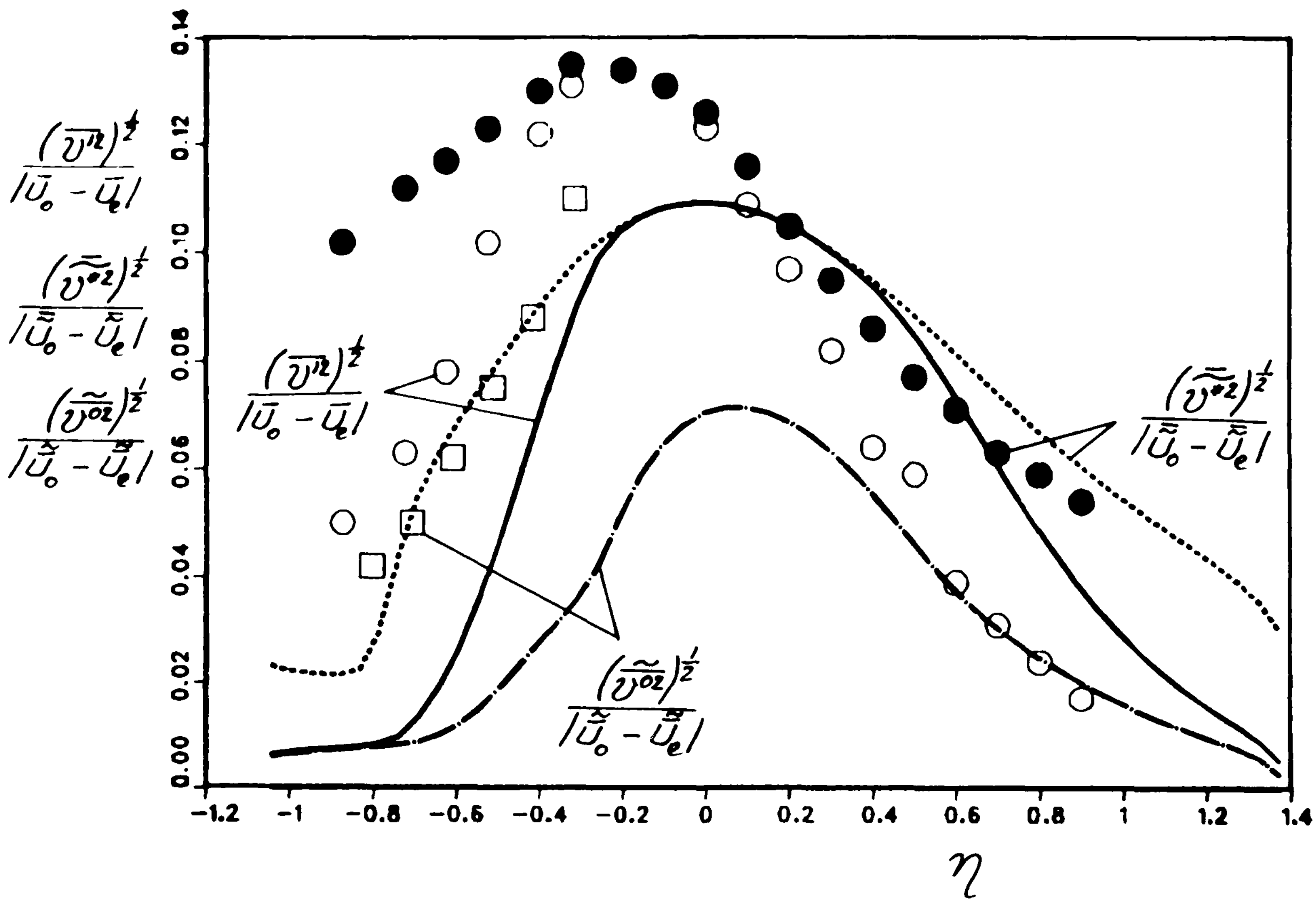
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
η
$\frac{(\overline{u^{*2}})^{\frac{1}{2}}}{|\bar{\bar{U}}_o - \bar{\bar{U}}_e|}$
$\frac{(\overline{v^{*2}})^{\frac{1}{2}}}{|\bar{\bar{U}}_o - \bar{\bar{U}}_e|}$
$\frac{(\overline{w^{*2}})^{\frac{1}{2}}}{|\bar{\bar{U}}_o - \bar{\bar{U}}_e|}$

Fig. 28



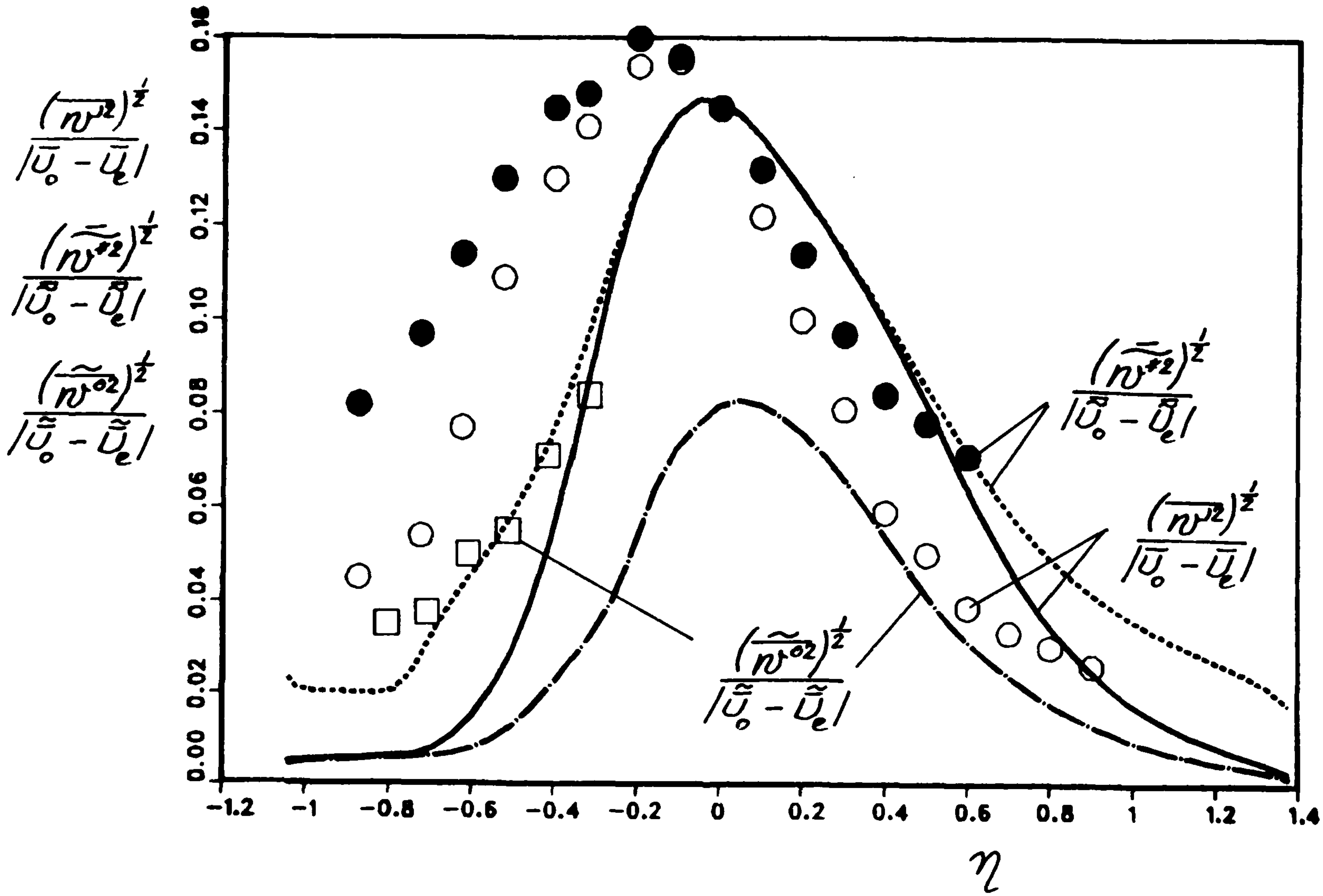
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
η

Fig. 29



$\frac{(\overline{v'^2})^{\frac{1}{2}}}{|\bar{U}_o - \bar{U}_e|}$
$\frac{(\overline{\tilde{v}^{*2}})^{\frac{1}{2}}}{|\bar{\bar{U}}_o - \bar{\bar{U}}_e|}$
$\frac{(\widetilde{v^{o2}})^{\frac{1}{2}}}{|\hat{U}_o - \hat{U}_e|}$
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
η

Fig. 30



$\frac{(\overline{v'^2})^{\frac{1}{2}}}{|\overline{U}_o - \overline{U}_e|}$
$\frac{(\overline{\tilde{v}^{*2}})^{\frac{1}{2}}}{|\bar{\bar{U}}_o - \bar{\bar{U}}_e|}$
$\frac{(\overline{\tilde{v}^{o2}})^{\frac{1}{2}}}{|\tilde{\bar{U}}_o - \tilde{\bar{U}}_e|}$
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
η

Multi-scale closure for turbulent shear flows

S. Byggstoyl and W. Kollmann

1. Introduction

Turbulence models based on a single time and length scale have successfully been used to calculate a number of equilibrium turbulent shear flows [1]. However, when the turbulence is not in equilibrium it is generally believed that it is necessary to include more than one length and time scale in the modelling. A multi-scale model was first developed by Hanjalic and Launder [2]. Based on the spectral energy transfer they intuitively constructed transport equations for different regions of the energy spectrum.

In this paper a new multiscale concept for modelling of turbulent flows is developed. The concept is based on statistics conditioned upon a positive scalar being inside some given intervals. Each interval (zone) has its own time and length scale and different statistics can be used for modelling the exact equations inside each zone. The dissipation of kinetic energy is taken as the scalar and then the dissipation term in the equation for turbulent kinetic energy in the zones is viewed as an independent scalar variable. The modelled transport equations for each zone have to be complemented with either the equation for the probability density function for the dissipation or the form of the Pdf has to be given explicitly. The paper is organized as follows: first the exact transport equations are derived and discussed, then the exact equations are modelled and solved for the case of isotropic turbulence and for a plane jet flow.

2. DERIVATION OF EXACT EQUATIONS

First the exact equations serving as basis for the multi-scale model are established. These include the transport equations for intermittency factor and conditioned first and second order moments. Some of the equations can be found in Ref. [3-6].

2.1 Intermittency factor

Let $\phi(\underline{x},t)$ be a fluctuating non-negative scalar satisfying the equation

$$\partial_t \phi + u_\alpha \partial_\alpha \phi = \partial_\alpha(\Gamma \partial_\alpha \phi) + S_\phi \qquad (2.1)$$

where S_ϕ expresses the production/destruction of the scalar ϕ (S_ϕ is assumed to be local in space and time but may be non-linear).

Consider a discretization of the range $\mathcal{R}(\phi)$

$$0 < \varphi_1 < \varphi_2 \quad \cdots \quad < \varphi_{N-1} < \varphi_N < \infty$$

and let the instantaneous flow field $\mathcal{D}$ be divided into overlapping zones $\mathcal{D}_i$, $i = 0, 1, \cdots, N+1$ defined as

$$\mathcal{D}_i \equiv \{ \underline{x} : \phi(\underline{x},t) > \varphi_i \}$$

It is then clear that $\mathcal{D}_{i+1} \subset \mathcal{D}_i$ for all i. In order to derive exact transport equations for the flow variables inside each zone, a sequence of indicator functions $I_k(\underline{x},t)$ corresponding to the sequence $\{\varphi_k\}_{k=0}^{N+1}$ is defined as

$$I_k(\underline{x},t) = \begin{cases} 1 & \phi(\underline{x},t) > \varphi_k \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$

The surface $S_k(\underline{x},t)$ corresponding to each φ_k in the range $R(\phi)$ is given by the equation

$$S_k(\underline{x},t) \equiv \phi(\underline{x},t) - \varphi_k = 0$$

Along this surface $dS_k = 0$ and the dynamics of the surface is described by the following equation

$$\partial_t S_k + v_\alpha^{S_k} \partial_\alpha S_k = 0 \tag{2.3}$$

where $v_\alpha^{S_k}$ is the instantaneous velocity of the surface given by

$$v_\alpha^{S_k} = d_t X_\alpha^{S_k}$$

where $\underline{X}^{S_k}$ is the position vector of the surface.

The velocity of the surface is further split into two parts

$$v_\alpha = v_\alpha^{S_k} + n_\alpha^k V^k \tag{2.4}$$

where v_α is the fluid velocity at the surface and V^k is the propagation velocity of the surface relative to the fluid. n_α^k is the unit normal vector on S_k defined as positive pointing into the domain $\mathcal{D}_k$.

The indicator function $I_k(\underline{x},t)$ is obviously constant along $S_k(\underline{x},t)$ and satisfies the equation

$$\partial_t I_k + v_\alpha^{S_k} \partial_\alpha I_k = 0 \tag{2.5}$$

Using (2.4) this tranforms into

$$\partial_t I_k + v_\alpha \partial_\alpha I_k = V^k n_\alpha^k \partial_\alpha I_k \tag{2.6}$$

The derivatives of the indicator function can be expressed as

$$\begin{aligned} \partial_\alpha I_k &= \partial_\alpha S_k \frac{\partial I_k}{\partial S_k} = \partial_\alpha S_k \, \delta(S_k) \\ \partial_t I_k &= \partial_t S_k \frac{\partial I_k}{\partial S_k} = \partial_t S_k \, \delta(S_k) \end{aligned} \tag{2.7}$$

where $\delta(\cdot)$ is the Dirac pseudo-function. Introducing (2.7) in (2.6) gives

$$\partial_t I_k + v_\alpha \partial_\alpha I_k = V^k |\nabla S_k| \delta(S_k)$$

By taking the mean value of this equation results in the equation for the intermittency factor $\gamma_k \equiv \langle I_k \rangle$

$$\partial_t \gamma_k + \partial_\alpha \langle I_k v_\alpha \rangle = \langle V^k |\nabla S_k| \delta(S_k) \rangle \tag{2.8}$$

At this point conditional mean and fluctuating velocities are introduced as

$$\begin{aligned} \bar{\tilde{v}}_\alpha^k &\equiv \frac{\langle I_k v_\alpha \rangle}{\gamma_k} \quad , \quad \overset{*}{v}_\alpha^k \equiv v_\alpha - \bar{\tilde{v}}_\alpha^k \\ \tilde{\tilde{v}}_\alpha^k &\equiv \frac{\langle (1 - I_k) v_\alpha \rangle}{1 - \gamma_k} \quad , \quad \overset{\circ}{v}_\alpha^k \equiv v_\alpha - \tilde{\tilde{v}}_\alpha^k \end{aligned} \tag{2.9}$$

where $\bar{\tilde{v}}_\alpha^k$ is the mean velocity, at $(\underline{x}, t)$ of fluid belonging to $\mathcal{D}_k$ and $\tilde{\tilde{v}}_\alpha^k$ is the mean velocity of the fluid which does not belong to $\mathcal{D}_k$.

By introducing this into (2.8) gives the following equation for the intermittency factor

$$\partial_t \gamma_k + \partial_\alpha(\bar{v}_\alpha^k \gamma_k) = \langle V^k |\nabla S_k| \delta(S_k) \rangle \tag{2.10}$$

The relative propagation velocity V^k can be expressed in terms of the discriminating scalar ϕ by using (2.3) and (2.4)

$$\partial_t \phi + \partial_\alpha(v_\alpha \phi) = V^k |\nabla \phi|$$

which is valid at the surface $S_k(\underline{x}, t)$.

By introducing (2.1) V^k can be expressed as

$$V^k = \left[\frac{1}{|\nabla \phi|}\left(\partial_\alpha(\Gamma \partial_\alpha \phi) + S_\phi\right)\right]_{S_k=0} \tag{2.11}$$

which shows that V^k is not bounded for $|\nabla \phi|_{S_k} \to 0$.

By introducing this into (2.10) the intermittency equation takes the following form

$$\partial_t \gamma_k + \partial_\alpha(\gamma_k \bar{v}_\alpha) = \langle [\partial_\alpha(\Gamma \partial_\alpha \phi) + S_\phi] \delta(\phi - \varphi_k) \rangle \tag{2.12}$$

The right hand side of this equation will be denoted by S_γ and expresses the mass entrainment per unit mass of fluid from zone $k-1$ into zone k. It can be shown that this term is positive or negative depending on the nature of the scalar and the threshold value φ_k.

In order to develop equations for the flow variables inside zones where the value of ϕ is inside some specified interval a non-overlapping decomposition must be used. This is done by defining $\mathcal{D}_{(k)}$ as

$$\mathcal{D}_{(k)} = \mathcal{D}_k - \mathcal{D}_{k-1}$$

which satisfy the requirement

$$\mathcal{D}_{(k)} \cap \mathcal{D}_{(l)} = \phi \quad \text{for } k \neq l .$$

The appropriate indicator function and intermittency factor for this decomposition re, respectively

$$I_{(k)} = I_k - I_{k+1} \quad , \quad \gamma_{(k)} = \gamma_k - \gamma_{k+1} \tag{2.13}$$

The conditional mean and fluctuating velocity for fluid belonging to $\mathcal{D}_{(k)}$ is then

$$v_\alpha^{(k)} = \frac{\langle (I_k - I_{k+1}) v_\alpha \rangle}{\gamma_k - \gamma_{k+1}} \quad , \quad v_\alpha^{*(k)} \equiv v_\alpha - v_\alpha^{(k)} \tag{2.14}$$

By using (2.14) and (2.12) gives the following equation for $\gamma_{(k)}$

$$\partial_t \gamma_{(k)} + \partial_\alpha (\gamma_{(k)} v_\alpha^{(k)}) = S_{\gamma_k} - S_{\gamma_{k+1}} \tag{2.15}$$

Note that the divergence of $v_\alpha^{(k)}$ is non zero and the relations $(\delta^*(S) \equiv |\nabla S| \delta(S))$

$$\partial_\alpha v_\alpha^{(k)} = -\partial_\alpha v_\alpha^{*(k)} = \frac{1}{\gamma_k - \gamma_{k+1}} \left[\langle v_\alpha^{*(k)} n_\alpha^{k} \delta^*(S_k) \rangle - \langle v_\alpha^{*(k)} n_\alpha^{k+1} \delta^*(S_{k+1}) \rangle \right]$$

hold, implying that $\partial_\alpha v_\alpha^{*(k)} \neq 0$ but does not fluctuate.

2.2 The Zone Averaged Momentum Equation

By multiplying the instantaneous momentum equation

$$\partial_t v_\alpha + v_\beta \partial_\beta v_\alpha = \partial_\beta (\nu \partial_\beta v_\alpha) - \frac{1}{\rho} \partial_\alpha p$$

with the indicator function $I_k(\underline{x}, t)$ and averaging gives, after some operations with the Dirac-pseudofunction , an equation for the zone conditioned mean velocity $v_\alpha^{(k)}$.

$$\partial_t (\gamma_k - \gamma_{k+1}) \overline{v}_\alpha^{(k)} + \overline{v}_\beta^{(k)} \partial_\beta (\gamma_k - \gamma_{k+1}) \overline{v}_\alpha^{(k)} = - \partial_\alpha (\gamma_k - \gamma_{k+1}) \frac{\overline{p}^{(k)}}{\rho}$$

$$+ \partial_\beta [\nu \partial_\beta (\gamma_k - \gamma_{k+1}) \overline{v}_\alpha^{(k)} - (\gamma_k - \gamma_{k+1}) \langle \overset{*}{v}_\alpha^{(k)} \overset{*}{v}_\beta^{(k)} \rangle] + H_\alpha^k - H_\alpha^{k+1}$$

The momentum fluxes through the iso-scalar surfaces S_k and S_{k+1} are given by

$$H_\alpha^k(\underline{v}, p) = \langle (v_\alpha V^k - \nu \partial_\beta v_\alpha n_\beta^k + \frac{p}{\rho} n_\alpha^k) \delta(S_k) \rangle - \partial_\beta \nu \langle v_\alpha n_\beta^k \delta(S_k) \rangle \tag{2.16}$$

and H_α^{k+1} follows by increasing the index k. The intermittency equation (2.15) allows rearrangement

$$\partial_t \overline{v}_\alpha^{(k)} + \overline{v}_\beta^{(k)} \partial_\beta \overline{v}_\alpha^{(k)} = \partial_\beta [\nu \partial_\beta \overline{v}_\alpha^{(k)} - \langle \overset{*}{v}_\alpha^{(k)} \overset{*}{v}_\beta^{(k)} \rangle] - \frac{1}{\rho} \partial_\alpha \overline{p}^{(k)}$$

$$- \langle \overset{*}{v}_\alpha^{(k)} \overset{*}{v}_\beta^{(k)} \rangle \partial_\beta \ln (\gamma_k - \gamma_{k+1}) + \frac{H_\alpha^k(\overset{*}{\underline{v}}^{(k)}, \overset{*}{p}^{(k)}) - H_\alpha^{k+1}(\overset{*}{\underline{v}}^{(k)}, \overset{*}{p}^{(k)})}{\gamma_k - \gamma_{k+1}} \tag{2.17}$$

Comparison of (2.17) with the equation for the unconditioned mean velocity $\overline{v}_\alpha$ reveals the appearance of two new term groups. The first group proportional to the zone-conditioned Reynolds-stress arises in the elimination of the intermittency factor on the left hand side. Its role can be elucidated for thin shear layers like jets, mixing layers and boundary layers and low values of k (say one) and $\gamma_{k+1} = 0$. Then can be seen that this source pushes the conditioned velocity profile further out than the unconditioned, hence increases the spreading rate. The second group is a collection of point statistical moments representing momentum transfer through the interface. Chapter three is devoted to the properties of this group.

2.3 Zone-averaged Reynolds-stress tensor

The transport equations for higher moments of zone-conditioned quantities can be obtained without difficulty using the properties of the indicator functions. The Reynolds-stress tensor satisfies for zone $(k, k+1)$ the following equation

$$
\begin{aligned}
\partial_t \tau^{(k)}_{\alpha\beta} + v^{(k)}_\gamma \partial_\gamma \tau^{(k)}_{\alpha\beta} = {} & \partial_\gamma F^{(k)}_{\alpha\beta\gamma} - \tau^{(k)}_{\alpha\gamma} \partial_\gamma v^{(k)}_\beta - \tau^{(k)}_{\beta\gamma} \partial_\gamma v^{(k)}_\alpha \\
& + \frac{1}{\rho} \langle p^{*(k)} (\partial_\alpha v^{*(k)}_\beta + \partial_\beta v^{*(k)}_\alpha) \rangle - 2\nu \langle \partial_\gamma v^{*(k)}_\alpha \partial_\gamma v^{*(k)}_\beta \rangle \\
& + (F^{(k)}_{\alpha\beta\gamma} + \nu \partial_\gamma \tau^{(k)}_{\alpha\beta}) \partial_\gamma \ln(\gamma_k - \gamma_{k+1}) + \frac{\tau^{(k)}_{\alpha\beta}}{\gamma_k - \gamma_{k+1}} \Big[\partial_\gamma \nu \partial_\gamma (\gamma_k - \gamma_{k+1}) \\
& - \langle V^k \delta^*(S_k) \rangle + \langle V^{k+1} \delta^*(S_{k+1}) \rangle \Big] + \frac{F^k_{\alpha\beta}(v^{*(k)}, p^{*(k)}) - F^{k+1}_{\alpha\beta}(v^{*(k)}, p^{*(k)})}{\gamma_k - \gamma_{k+1}}
\end{aligned}
\qquad (2.18)
$$

The Reynolds-stress is defined by

$$
\tau^{(k)}_{\alpha\beta} = \langle v^{*(k)}_\alpha v^{*(k)}_\beta \rangle
$$

and the turbulent flux of $\tau^{(k)}_{\alpha\beta}$ by

$$
F^{(k)}_{\alpha\beta\gamma} \equiv \nu \partial_\gamma \tau^{(k)}_{\alpha\beta} - \langle v^{*(k)}_\alpha v^{*(k)}_\beta v^{*(k)}_\gamma \rangle - \delta_{\alpha\gamma} \frac{1}{\rho} \langle v^{*(k)}_\beta p^{*(k)} \rangle - \delta_{\beta\gamma} \frac{1}{\rho} \langle v^{*(k)}_\alpha p^{*(k)} \rangle
$$

Conditional statistics introduce two new term groups as in the mean velocity equation (2.17). The first group, arising from the elimination of the intermittency factors on the left hand side, can be recast as

$$(F^{(k)}_{\alpha\beta\sigma} + \nu\partial_\sigma \tau^{(k)}_{\alpha\beta})\partial_\sigma \ln(\gamma_k - \gamma_{k+1}) + \frac{\tau^{(k)}_{\alpha\beta}}{\gamma_k - \gamma_{k+1}}[\partial_\sigma \nu \partial_\sigma \tau^{(k)}_{\alpha\beta} - \langle V^k \delta(S_k)\rangle$$

$$+ \langle V^{k+1}\delta(S_{k+1})\rangle] = F^{(k)}_{\alpha\beta\sigma}\partial_\sigma \ln(\gamma_k - \gamma_{k+1}) - \tau^{(k)}_{\alpha\beta}\frac{D^{(k)}}{Dt}\ln(\gamma_k - \gamma_{k+1})$$

+ viscous + Divergence terms,

where $\frac{D^{(k)}}{Dt} \equiv \partial_t + v^{(k)}_\sigma \partial_\sigma$.

This term group can be expected to provide additional transport of $\tau^{(k)}_{\alpha\beta}$ thus changing spreading rates of $\tau^{(k)}_{\alpha\beta}$ -profiles in thin shear layers. A more detailed estimation of its effect is however difficult due to the lack of experimental information on the intermittency sources. The second group of point-statistical correlations defined by

$$F^k_{\alpha\beta}(\underline{v},p) \equiv \langle v_\alpha v_\beta V^k \delta(S_k)\rangle + \frac{1}{\rho}\langle p(v_\beta n^k_\alpha + v_\alpha n^k_\beta)\delta(S_k)\rangle$$

$$- \nu\langle n^k_\sigma \partial_\sigma(v_\alpha v_\beta)\delta(S_k)\rangle - \partial_\sigma(\nu\langle v_\alpha v_\beta n^k_\sigma \delta(S_k)\rangle) \quad (2.19)$$

will be discussed in chapter three.

The equation for the kinetic energy of turbulence $k^{(k)}$ for the k^{th} -zone deserves special attention for $\phi \equiv \varepsilon$. It follows from (2.18) as

$$\partial_t k^{(k)} + v^{(k)}_\alpha \partial_\alpha k^{(k)} = \partial_\alpha F^{(k)}_{\beta\beta\alpha} - \tau^{(k)}_{\alpha\beta}\partial_\beta v^{(k)}_\alpha - \varepsilon^{(k)}$$

$$+ (F^{(k)}_{\alpha\alpha\beta} + \nu\partial_\beta k^{(k)})\partial_\beta \ln(\gamma_k - \gamma_{k+1})$$

$$+ \frac{k^{(k)}}{\gamma_k - \gamma_{k+1}}[\partial_\alpha \nu \partial_\alpha(\gamma_k - \gamma_{k+1}) - \langle V^k \delta(S_k)\rangle + \langle V^{k+1}\delta(S_{k+1})\rangle]$$

$$+ \frac{F^k_{\alpha\alpha} - F^{k+1}_{\alpha\alpha}}{\gamma_k - \gamma_{k+1}}$$

The dissipation rate conditioned for zone k is obviously bounded by φ_k and φ_{k+1}, hence

$$\varepsilon^{(k)} = \frac{1}{2}(\varphi_k + \varphi_{k+1}) + O(\varphi_k - \varphi_{k+1})$$

If the interval length $|\varphi_k - \varphi_{k+1}|$ is small enough, this does not constitute an unknown correlation (for which an equation could be derived) but an additional discrete independent variable. The zone-conditioned moments can be considered therefore functions of $\underline{x}, t$ and φ. If we assume for the moment that $F_{\alpha\alpha}^{k} > 0$ and $F_{\alpha\alpha}^{k+1} > 0$ we see that several inverse time scales can be formed for the k^{th} zone:

Energy input from zone $k-1$: $\tau_{in}^{-1} = \dfrac{2}{k^{(k)} + k^{(k-1)}} \cdot \dfrac{F_{\alpha\alpha}^{k}}{\gamma_k - \gamma_{k+1}}$

Energy output to zone $k+1$: $\tau_{out}^{-1} = \dfrac{2}{k^{(k)} + k^{(k+1)}} \dfrac{F_{\alpha\alpha}^{k+1}}{\gamma_k - \gamma_{k+1}}$

Energy transfer through zone k : $\tau_{tr}^{-1} = \dfrac{1}{k^{(k)}} \cdot \dfrac{|F_{\alpha\alpha}^{k} - F_{\alpha\alpha}^{k+1}|}{\gamma_k - \gamma_{k+1}}$

Average scale for zone k : $\tau^{-1} = \dfrac{\varphi_k + \varphi_{k+1}}{2\, k^{(k)}}$

The input and output scales make sense however only for finite intervals $\Delta\varphi_k$. Hence only the transfer scale and the average scale will be used in the following together with the corresponding length scales formed with appropriate powers of k.

2.4 Relation to unconditional moments

Unconditional moments can be recovered from the sequence of conditional moments via local relations following from the definitions given at the beginning of this chapter. Noting that $\gamma_0 = 1$ and $\gamma_{N+1} = 0$ we obtain for the unconditional mean value $\bar{\psi}$ of a fluctuating quantity ψ

$$\bar{\psi} = \sum_{k=0}^{N} \psi^{(k)} (\gamma_k - \gamma_{k+1}) \tag{2.20}$$

for the special case $N=1$ we note that $\psi^{(0)} = \tilde{\bar{\psi}}$ and $\psi^{(1)} = \bar{\bar{\psi}}$ and then reduces (2.20) to the well-known relation

$$\bar{\psi} = \gamma_1 \bar{\bar{\psi}} + (1-\gamma_1)\tilde{\bar{\psi}}$$

Furthermore is

$$\sum_{k=0}^{N} (\gamma_k - \gamma_{k+1}) = 1$$

and the terms in the sum are nonnegative due to monotonicity of the intermittency factors. For second order moments follows similarly

$$\overline{\phi'\psi'} = \sum_{k=0}^{N} \langle \phi^{*(k)} \psi^{*(k)} \rangle (\gamma_k - \gamma_{k+1}) + \sum_{k=0}^{N} \phi^{(k)} \psi^{(k)} (\gamma_k - \gamma_{k+1})$$

$$- \sum_{k=0}^{N} \sum_{L=0}^{N} \phi^{(k)} \psi^{(L)} (\gamma_k - \gamma_{k+1})(\gamma_L - \gamma_{L+1}) \tag{2.21}$$

For the special case N = 1 it reduces to the relation

$$\overline{\phi'\psi'} = \gamma_1 \langle \phi^{*(1)} \psi^{*(1)} \rangle + (1-\gamma_1) \langle \phi^{*(0)} \psi^{*(0)} \rangle + \gamma_1 (1-\gamma_1)(\phi^{(1)} - \phi^{(0)})(\psi^{(1)} - \psi^{(0)})$$

The formulae for higher order correlations can be derived without difficulty.

3. Transport Through Iso-Scalar Interfaces

Zone-conditioned moment equations contain a term group accounting for the transport effects through the iso-scalar surface represented by the implicit equation

$$S_k(\underline{x},t) = \phi(\underline{x},t) - \varphi_k = 0$$

This term group appears in the intermittency factor equations as

$$I_k^{\,0} \equiv \langle V^k \delta^*(S_k) \rangle - \langle V^{k+1} \delta^*(S_{k+1}) \rangle \tag{3.1}$$

and in the equation for moments of order one and higher as

$$I_\alpha^{\,k} \equiv \frac{H_\alpha^{\,k} - H_\alpha^{\,k+1}}{\gamma_k - \gamma_{k+1}} \tag{3.2}$$

provided the demoninator is nonzero. $H_\alpha^k(\underline{v},p)$ denotes a collection of point-statistical correlations of the arguments velocity and pressure with the relative progressive velocity V^k and the normal vector n_α^k of the iso-scalar surface $S_k = 0$.

3.1 Source terms of intermittency

The term group (3.1) represents the transport of the iso-scalar surface itself and contains productive and destructive effects. This can be seen by considering the limit $\Delta\varphi_k \to 0$. First we notice that

$$\gamma_k - \gamma_{k+1} = \langle (I_k - I_{k+1}) \rangle = \int_{\varphi_k}^{\infty} d\varphi \, P(\varphi) - \int_{\varphi_k + \Delta\varphi_k}^{\infty} d\varphi \, P(\varphi)$$

where $P(\varphi;\underline{x},t)$ is the one-point pdf of the discriminating scalar ϕ. For $\Delta\varphi_k > 0$ but sufficiently small and $P(\varphi)$ sufficiently smooth follows

$$\gamma_k - \gamma_{k+1} = \Delta\varphi_k P(\varphi_k) + h.o.t. \tag{3.3}$$

Furthermore follows from the definition (2.14)

$$\lim_{\Delta\varphi_k \to 0} \underline{v}^{(k)} = \langle \underline{v} / \phi = \varphi_k \rangle \tag{3.4}$$

which is the mean of $\underline{v}$ subject to the condition that the scalar ϕ assumes at the same (fixed) location the value φ_k. The relations (3.3) and (3.4) are the primary tools in analyzing the sources (3.1). Making the φ-interval sufficiently small and applying (3.3) and (3.4) to (2.14) we obtain

$$\partial_t(\Delta\varphi_k P) + \partial_\alpha(\langle v_\alpha / \phi = \varphi_k \rangle \Delta\varphi_k P) = \langle V^k \delta(S_k)\rangle - \langle V^{k+1}\delta(S_{k+1})\rangle + O(\Delta\varphi_k^2)$$

Dividing the $\Delta\varphi_k$ and letting $\Delta\varphi_k \to 0$ leads to

$$\partial_t P + \partial_\alpha(\langle v_\alpha / \phi = \varphi_k \rangle P) = \lim_{\Delta\varphi_k \to 0} \frac{1}{\Delta\varphi_k}\left[\langle V^k \delta(S_k)\rangle - \langle V^{k+1}\delta(S_{k+1})\rangle\right] \tag{3.5}$$

The limit on the right hand side can be evaluated by direct derivation of the pdf-equation. Defining the fine-grained pdf $\hat{P}$ by [7]

$$\hat{P} \equiv \delta(\phi - \varphi) \quad , \quad P = \langle \hat{P} \rangle$$

the pdf equation follows [8] as

$$\partial_t P + \partial_\alpha(\langle v_\alpha / \phi = \varphi \rangle P) = -\partial^2_{\varphi\varphi}\langle \varepsilon^s \hat{P} \rangle - \dots$$

Equating the right hand sides of (3.5) and (3.6) ...

result

$$\lim_{\Delta\varphi_k \to 0} \frac{1}{\Delta\varphi_k}\left[\langle V^k \delta(S_k)\rangle - \langle V^{k+1}\delta(S_{k+1})\rangle\right] = - \dots$$

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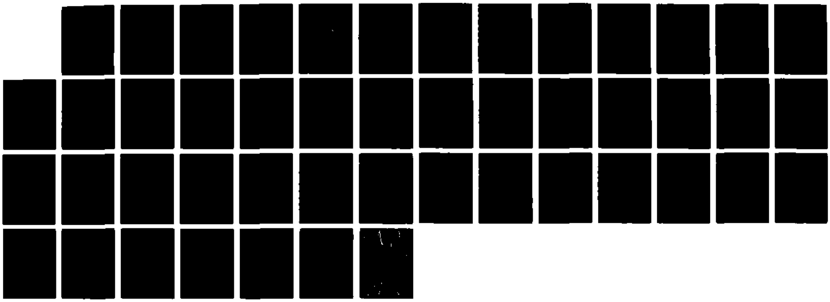

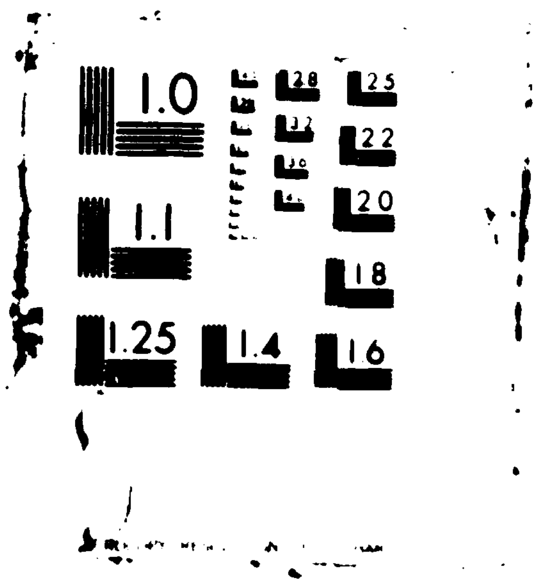
1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
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1.8
1.25
1.4
1.6

Note that $\mathcal{E}^s$ is defined as

$$\mathcal{E}^s \equiv \Gamma \nabla\phi . \nabla\phi$$

From (3.7) follows that the limit on the left hand side exists and therefore

$$\partial_\varphi \langle \nu \partial^\varphi(S) \rangle = \partial_\varphi (S(\varphi) P) + \partial^2_{\varphi\varphi} \langle \mathcal{E}^s \hat{P} \rangle$$

and finally

$$\langle \nu \partial^\varphi(S) \rangle = S(\varphi) P(\varphi) + \partial_\varphi \langle \mathcal{E}^s \hat{P} \rangle + const. \qquad (3.8)$$

Equation (3.8) allows a detailed discussion of the intermittency source/sink term for the case $P(\varphi)$ close to log-normal. First we note that

$$\langle \mathcal{E}^s \hat{P} \rangle \geq 0$$

and therefore will $\partial_\varphi \langle \mathcal{E}^s \hat{P} \rangle$ change sign at least once because $\langle \mathcal{E}^s \hat{P} \rangle \to 0$ as $\varphi \to \infty$ and $\langle \mathcal{E}^s \hat{P} \rangle(0) = 0$. The second derivative in (3.7) will be negative around the mean $\langle \phi \rangle$ of P [8], hence is (setting $S = 0$ for the moment) $\partial_\varphi \langle \nu \partial^\varphi(S) \rangle \geq 0$. For values φ far away from $\langle \phi \rangle$ the second derivative [illegible] will be positive making $\partial_\varphi \langle \nu \partial^\varphi(S) \rangle \leq 0$ if $P(\varphi)$ is smooth with a single peak near $\langle \phi \rangle$. If we specialize the discriminating [illegible] $\phi \equiv \varepsilon$, the dissipation rate of turbulent kinetic energy, and consider a region in the turbulent flow field where $\langle \varepsilon \rangle \gg 0$, then [illegible] that due to the dissipation of ε

$$[illegible] > 0$$

[illegible] ψ_a [illegible] $\langle \varepsilon \rangle$ and [illegible] $\langle \varepsilon \rangle$ [illegible]

$$[illegible] \leq 0$$

[illegible] ψ_a [illegible] $\langle \varepsilon \rangle$ [illegible] statements [illegible] the [illegible]

ψ_a [illegible] that

$$\langle \nu^k \delta^*(s_k) \rangle > 0$$

for $\varphi_k < \langle \varepsilon \rangle$ and

$$\langle \nu^k \delta^*(s_k) \rangle < 0$$

for $\varphi_k > \langle \varepsilon \rangle$. The scalar dissipation terms appearing in (3.7) and (3.8) are sketched in fig. 1 for the case of $P(\varphi)$ being close to log-normal form.

The effect of source $S(\varphi)$ of the dissipation rate ε on the intermittency factor γ_k becomes apparent from (3.8). If $S(\varphi_k) > 0$ then γ_k is produced, but $S(\varphi_k) > 0$ implies at the same time a shift of $P(\varphi)$ towards higher values of ϕ and the growth of γ_k is changed according to the shape of P.

The shape of the γ_k-profiles can be inferred from (3.5), (3.8) and the boundary conditions with respect to the φ-axis.

$$\gamma_0 = 1 \quad \text{for } \varphi = 0 \quad , \quad \gamma_{N+1} = 0 \quad \text{for } \varphi = \varphi_{N+1}$$

Equation (3.8) shows furthermore that (provided the constant in (3.8) is zero) the realizability condition

$$0 \leq \gamma_k \leq 1 \quad \text{for } k = 0, 1, \ldots, N+1$$

is satisfied, because the right hand side is the source term of the equation for (integrate (3.6))

$$\psi(\varphi) \equiv 1 - \int_0^{\varphi} d\varphi' P(\varphi')$$

which satisfies $0 \leq \psi \leq 1$.

For small φ_K (close to the boundary $\varphi_0 = 0$ where $\gamma_K = 1$) the profile of γ_K will be close to $\gamma_0 = 1$ if the turbulent Reynolds-number is high, because the pdf of ε will be small for small values of φ and be concentrated at high values of φ and therefore $S(\varphi)$ and $\langle \varepsilon^s \beta \rangle$ will be small too. For high values $\varphi_K \leq \langle \varepsilon \rangle_{max}$ the profile of γ_K will peak near $\langle \varepsilon \rangle_{max}$ where the source of γ_K may still be positive. Further away from the location of $\langle \varepsilon_{max} \rangle$ where $\langle \varepsilon \rangle \ll \langle \varepsilon \rangle_{max}$ the source of γ_K will be negative (see fig. 1). The sequence of intermittency factors is however monotonically decreasing everywhere in $\underline{x}$-space.

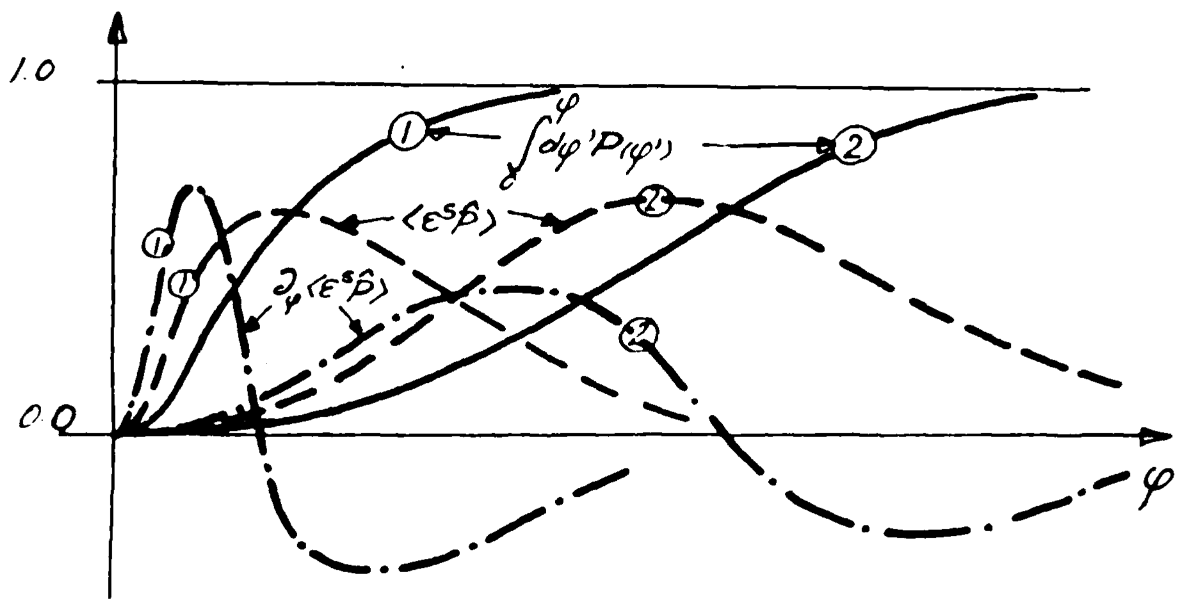


Figure 1 - Distribution function and scalar dissipation function for two different spatial locations with $\langle \phi \rangle_2 > \langle \phi \rangle_1$.

These properties of γ_κ are summarized in fig. 2 for the example of a thin shear layer

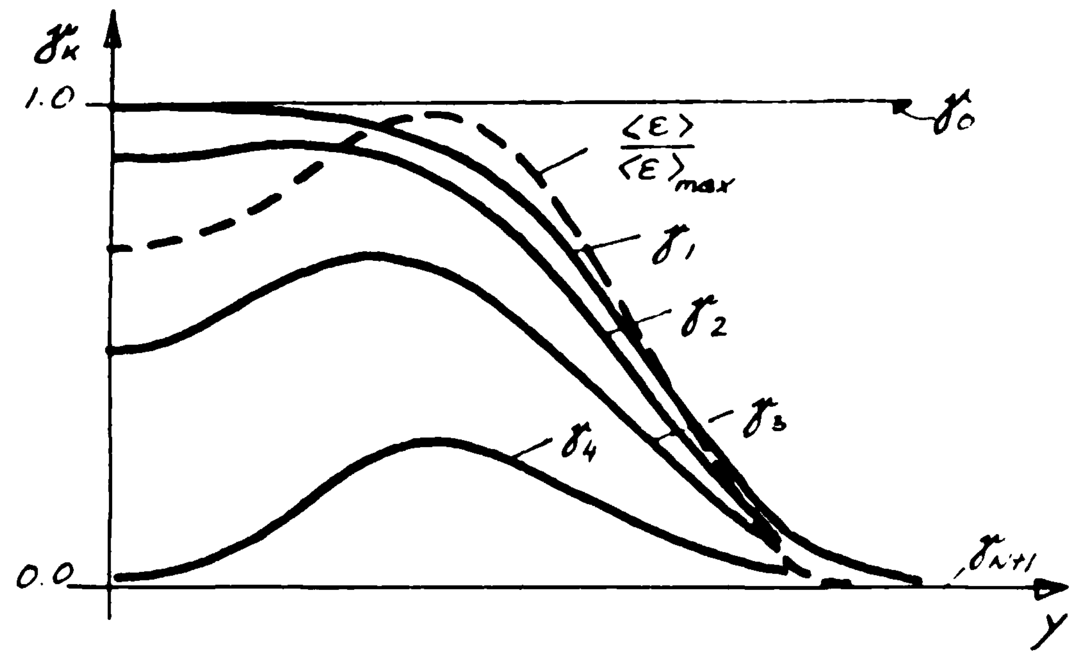


Figure 2 - Intermittency factors γ_κ for plane jet and N=4.

It should be noted that for $\Delta\varphi_\kappa$ sufficiently small the profile of γ_1 corresponds to the profile of the classical intermittency factor which distinguishes between turbulent and non-turbulent states of the flow. A large amount of experimental data is available for this case (references given in [6]).

The relative progresson velocity V^κ of the iso-scalar surface $S_\kappa = 0$ was derived in sec. 2.1 as

$$V(\varphi) = |\nabla\phi|^{-1}\left[\partial_\alpha(\Gamma\partial_\alpha\phi) + S(\phi)\right]_{\phi=\varphi} \qquad (3.9)$$

This can be applied to (3.9) and leads to a new expression of the scalar dissipation term of the pdf-equation for a fluctuating scalar. We find

$$\langle \varepsilon^s \hat{P} \rangle = \int_{-\infty}^{\varphi} d\varphi' \langle V |\nabla\phi| \delta(\phi - \varphi') \rangle$$

Hence is the scalar dissipation of P at φ the integral of the relative progression speed of all iso-scalar surfaces with $\varphi' \leq \varphi$. Equation (3.9) shows now that the instantaneous values of V are not bounded and the amount of scalar dissipation of P at φ depends on the Pdf of ϕ.

$$\langle \varepsilon^s \hat{P} \rangle = \int_{-\infty}^{\varphi} d\varphi' \langle \partial_\alpha (\Gamma \partial_\alpha \phi) \delta(\phi - \varphi') \rangle \qquad (3.10)$$

3.2 Source terms for zone-conditioned mean velocity

The term group (2.16) represents the transfer of momentum through the iso-scalar surface $S_\kappa = 0$ and contains correlations of zone-conditional velocity with the progression speed of the surface, the zone-conditional pressure with the fluctuating normal vector of the interface and viscous interactions of velocity and normal vector. The limit $\Delta\varphi_\kappa \to 0$ reveals again the effect of this group on the zone-conditional moment concerned. With (3.3) we can write

$$I_\alpha^k = \frac{H_\alpha^k - H_\alpha^{k+1}}{\Delta\varphi_k \, P(\varphi_k)} + h.o.t$$

The terms constituting H_α^κ contain a viscous contribution of the form

$$\partial_\beta (\nu \langle u_\alpha n_\beta^k \delta^*(S_\kappa) \rangle)$$

which is negligible compared to the other terms for high Re_t-numbers because it scales with the mean field. Hence it will be neglected and H_α^K appears to be

$$H_\alpha^k(\underline{v}^{*(k)},p^{*(k)}) \cong P(\varphi_k)\langle[v_\alpha^{*(k)}V^k+\frac{1}{\rho}p^{*(k)}n_\alpha^k-\nu\partial_\beta v_\alpha^{*(k)}n_\beta^k]/\phi=\varphi_k\rangle$$

Then follows for I_α^k

$$I_\alpha^k = \frac{P(\varphi_k)\langle[\cdots]/\phi=\varphi_k\rangle - P(\varphi_k+\Delta\varphi_k)\langle[\cdots]/\phi=\varphi_k+\Delta\varphi_k\rangle}{\Delta\varphi_k\, P(\varphi_k)}$$

The fluctuating components are in the limit $\Delta\varphi_k \to 0$

$$v_\alpha^{*(k)} = v_\alpha - \langle v_\alpha/\phi=\varphi_k\rangle \qquad etc$$

and I_α^k turns out as

$$I_\alpha^k = -\frac{1}{P(\varphi_k)}\,\partial_\varphi\{P(\varphi_k)\langle[v_\alpha^{*(k)}V^k+\frac{1}{\rho}p^{*(k)}n_\alpha^k-\nu n_\beta^k\partial_\beta v_\alpha^{*(k)}]/\phi=\varphi_k\rangle\} \qquad (3.11)$$

provided $P(\varphi_k)>0$. The terms I_α^k do not contribute to the overall (unconditional) momentum balance. This can be seen from the expectation of I_α^k

$$\langle I_\alpha^k\rangle = \int_0^\infty d\varphi\, P(\varphi)\, I_\alpha^k(\varphi)$$

with (2.32) we obtain

$$\langle I_\alpha^k\rangle = P(\varphi)\langle[\cdots]/\phi=\varphi\rangle\Big|_{\varphi=0}^{\varphi=\infty} = 0$$

Hence transfer the point-statistical terms I_α^k momentum along the φ-axis without affecting the unconditional momentum balance. Since all zone-conditioned mean velocities must satisfy the same boundary conditions, momentum is transferred from the boundaries to all zones. For

small values of ϕ the zone-conditioned shear stress can be expected to be small and will therefore be insufficient for momentum transport to keep the corresponding zone-mean velocity field stable. Hence must the point-statistical term group I_α^k transfer enough momentum out of these zones to ensure stability. As φ increases the Reynolds-stress in these zones will become more important for momentum transfer and I_α^k will decrease and change sign. For high values φ the role of I_α^k will be opposite to its effect for low φ.

3.3 Source terms for zone-conditioned Reynolds-stresses

The term group (2.19) represents the transfer of stress across the iso- scalar surface $S_\alpha = O$. Its properties are similar to the corresponding group in the momentum equations. In the limit $\Delta\varphi_k \to O$ we find

$$I_{\alpha\beta}^k = -\frac{1}{P(\varphi)}\partial_\gamma\Big\{P(\varphi)\Big\langle\Big[\overset{*(k)}{v_\alpha}\overset{*(k)}{v_\beta}V^k + \frac{1}{\rho}\overset{*(k)}{p}\big(\overset{*(k)}{v_\beta}n_\alpha^k + \\ + \overset{*(k)}{v_\alpha}n_\beta^k\big) - \nu\partial_\gamma\big(\overset{*(k)}{v_\alpha}\overset{*(k)}{v_\beta}\big)n_\gamma^k\Big]\Big|\phi = \varphi_k\Big\rangle\Big\} \qquad (3.12)$$

where the last viscous term in (12) is again negligible. The unconditional expectation of $I_{\alpha\beta}^k$ is zero

$$\langle I_{\alpha\beta}^k\rangle = \int_0^\infty d\varphi\, P(\varphi)\, I_{\alpha\beta}^k(\varphi) = 0$$

indicating that the point-statistical group (2.19) transfers stress and in particular kinetic energy along the φ-axis without affecting the unconditional balance. Furthermore must $I_{\alpha\beta}(\varphi)$ change sign at least once with respect to φ. If the choice $\phi = \epsilon$ is considered, the high values φ of ε correspond to the small scales of the turbulent motion.

For high turbulent Re-numbers local isotropy can be assumed for this range and small normal stress levels. Then follows that there is little production of zone-Reynolds-stress due to mean strain rate and most production will be supplied by the transfer term (3.12), which is therefore positive for high φ-values and $\alpha = \beta$ and negative for small φ-values.

3.4 Multiscale closure models

The first step in the development of mutli-scale models based on conditional statistics is the choice of the discriminating scalar ϕ. The most natural choice is the dissipation of turbulent kinetic energy because this quantity appears in the equation for turbulent kinetic energy (and in the equation for the individual normal stresses). If the turbulence field is close to equlibrium it is probably not necessary to solve an equation for the PdF, but instead use a prescribed Pdf expressed in terms of some of its moments. If ε is taken as the discriminating scalar then the log-normal distribution may be a good choice [9]. In a non-equilibrium situations it is necessary to solve a modelled equation for the intermittency factors γ_α or to solve the Pdf equation.

4. Closure of the Multiscale Equations

In this chapter closure of the multiscale equations is discussed and a simple closure model is suggested. The model is applied to decaying isotropic turbulence and to a plane jet.

4.1 Multiscale Model for Decaying Isotropic Turbulence

In the case of isotropic turbulence the equation for zone conditioned turbulent kinetic energy simplifies to

$$\partial_t k^{(k)} = -\varepsilon^{(k)} + \frac{k^{(k)}}{\gamma_k - \gamma_{k+1}}\left[\langle V^{k+1}\partial \gamma^*(S_{k+1})\rangle - \langle V^k \partial \gamma(S_k)\rangle\right]$$

$$+ \frac{F_{\alpha\alpha}^{k} - F_{\alpha\alpha}^{k+1}}{\gamma_k - \gamma_{k+1}} \qquad (4.1)$$

By using (3.8) and (3.12) this equation transforms into (by letting $\Delta\varphi = \Delta\varepsilon \to 0$).

$$\partial_t (k^{(k)} P(\varepsilon)) = -\partial_\varepsilon F - \varepsilon^{(k)} P(\varepsilon) \qquad (4.2)$$

where $F = -P(\varepsilon) I_{\alpha\alpha}^{k}$ (See (3.12))

Equation (4.2) has the same form as the equation for the energy spectrum for isotropic turbulence. This is seen by first integrating over all values of ε, then the following equation is obtained.

$$d_t \langle k \rangle = -\langle \varepsilon \rangle$$

which is the equation for kinetic energy for decaying isotropic turbulence.

By integrating (4.2) from 0 to some ε_1 yields

$$\partial_t \tilde{k}_1 = -F(\varepsilon_1) - \tilde{\varepsilon}_1 \qquad (4.3)$$

where $\tilde{k}_1 = \int_0^{\varepsilon_1} d\varepsilon\, k^{(k)}_{(\varepsilon)} P(\varepsilon)$, $\tilde{\varepsilon}_1 = \int_0^{\varepsilon_1} d\varepsilon\, \varepsilon P(\varepsilon)$.

the term $-F_1(\varepsilon_1)$ expresses the net energy transport from zones with $\varepsilon \le \varepsilon_1$ to zones with $\varepsilon > \varepsilon_1$, this is illustrated in Fig. 3.

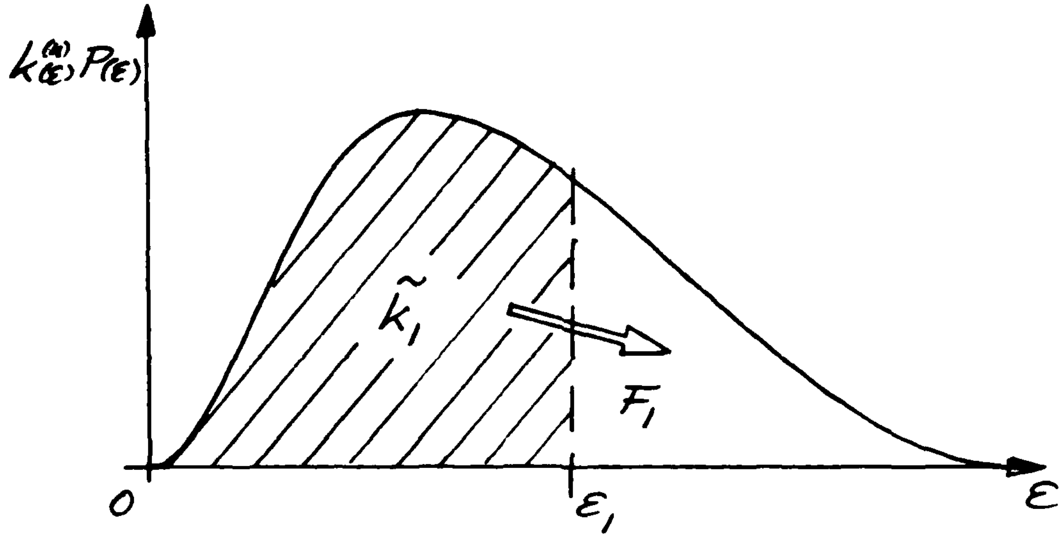


Figure 3. - Schematic illustration of the energy transport (eq.(4.3))

As seen from (4.2) the Pdf of ε, $P(\varepsilon)$ is needed. This can be obtained by a direct solution of a modelled transport equation for the pdf or by prescribing a pdf which depends on some of its moments. Both of these methods will be discussed next.

4.1.1 *The Transport Equation for $P(\varepsilon)$.*

The exact transport equation for $P(\varepsilon)$ can easily be derived from the Navier Stokes equation and its form is given by (3.6). The equation for $P(\varepsilon)$ is derived in Appendix 1 and for the case of isotropic turbulence it is given by

$$\partial_t P(\varepsilon) = -\partial^2_{\varepsilon\varepsilon}\langle \nu \nabla\tilde{\varepsilon}\cdot\nabla\tilde{\varepsilon}\,\hat{P}\rangle - 2\nu\,\partial_\varepsilon\langle [\partial^2_{\alpha\beta}p\cdot\partial_\beta u_\alpha$$

$$+ \nu\,\partial^2_{\beta\gamma}u_\alpha\,\partial^2_{\beta\gamma}u_\alpha - \partial_\gamma u_\alpha\,\partial_\beta u_\alpha\,\partial_\beta u_\gamma]\,\hat{P}\rangle \qquad (4.4)$$

where ε is the probability space variable corresponding to the scalar

$$\tilde{\varepsilon} = \nu\,\partial_\beta u_\alpha\,\partial_\beta u_\alpha$$

The scalar dissipation term is modelled according to [8]

$$-\partial^2_{\varepsilon\varepsilon}\langle \nu \nabla\hat{\varepsilon}\cdot\nabla\hat{\varepsilon}\hat{P}\rangle \cong \frac{A}{\tau}\left\{2\int_0^{\varepsilon} d\varepsilon' \int_{\varepsilon}^{\infty} d\varepsilon'' P(\varepsilon')P(\varepsilon'')T(\varepsilon';\varepsilon''/\varepsilon) - P(\varepsilon)\right\} \tag{4.5}$$

where the transition probability T is given as

$$T(\varepsilon';\varepsilon''/\varepsilon) = \begin{cases} \dfrac{1}{|\varepsilon''-\varepsilon'|} & \text{for } \varepsilon' \le \varepsilon \le \varepsilon'' \text{ or } \varepsilon'' \le \varepsilon \le \varepsilon' \\ 0 & \text{otherwise} \end{cases} \tag{4.6}$$

and the timescale τ as

$$\tau = \frac{\langle k\rangle}{\langle\varepsilon\rangle} \tag{4.7}$$

It is easily shown that the expression (4.5) decreases the variance of the Pdf without affecting the mean, therefore the last term on the right hand side of (4.4) must be responsible for the decay of $\langle\varepsilon\rangle$. This term is modelled as

$$-2\nu\,\partial_{\varepsilon}\langle[\partial^2_{\alpha\beta}p\,\partial_{\beta}u_{\alpha} + \nu\partial^2_{\beta\gamma}u_{\alpha}\partial^2_{\beta\gamma}u_{\alpha} - \partial_{\gamma}u_{\alpha}\partial_{\beta}u_{\alpha}\partial_{\beta}u_{\gamma}]\hat{P}\rangle \cong$$

$$\cong \frac{C_2}{\tau}\partial_{\varepsilon}(\varepsilon P) \tag{4.8}$$

which represent a convection of the Pdf towards $\varepsilon = 0$ (for $C_2 > 0$) with convection velocity $v_{\varepsilon} \equiv C_2\dfrac{\varepsilon}{\tau}$. Because this velocity is not uniform but depends on ε the higher order moments will also be influenced by this term.

The modelled transport equation for $P(\varepsilon)$ is then

$$\partial_t P = \frac{A}{\tau}\left\{2\int_0^{\varepsilon} d\varepsilon' \int_{\varepsilon}^{\infty} d\varepsilon'' P(\varepsilon')P(\varepsilon'') T(\varepsilon';\varepsilon''/\varepsilon) - P\right\} + \frac{C_2}{\tau}\partial_{\varepsilon}(\varepsilon P) \tag{4.9}$$

By multiplying (4.9) with ε and ε^2 respectively and integrating over ε transport equations for $\langle\varepsilon\rangle$ and $\langle\varepsilon'^2\rangle$ are found. They are

$$d_t\langle\varepsilon\rangle = -C_2\frac{\langle\varepsilon\rangle}{\tau} \tag{4.10}$$

$$d_t\langle\varepsilon'^2\rangle = -\frac{1}{\tau}\left(\frac{A}{3} + 2C_2\right)\langle\varepsilon'^2\rangle \tag{4.11}$$

Equation (4.10) is the same as the $\langle\varepsilon\rangle$ equation in the model of turbulence (in the case of decaying isotropic turbulence) and the constant C_2 is therefore put equal to $C_2 = 1.92$. It is known from experiments that the ratio $\langle\varepsilon'^2\rangle/\langle\varepsilon\rangle$ increases with increasing Reynolds-number of turbulence, then by solving the $\langle k\rangle - \langle\varepsilon\rangle$ model together with (4.11) it can be shown (See Appendix 2) that this constant A can be given when comparing (4.11) with Kolmogorov's modified equilibrium theory [10] to be discussed next.

4.1.2 Prescribed form of the Pdf

In order to take into account the intermittent behaviour of the dissipation Kolmogorov [10] and Obukov [11] refined the "universal similarity hypothesis" [12] by postulating that the dissipation of kinetic energy was log-normally distributed with a variance dependent on the Reynolds number. More specifically they postulated that the Pdf of the dissipation was given by

$$P(\varepsilon) = \frac{1}{\sqrt{2\pi}\,\beta\varepsilon}\exp\left[-\left(\frac{\ln\varepsilon - m}{\sqrt{2}\,\beta}\right)^2\right] \tag{4.12}$$

where $m = \langle\ln\varepsilon\rangle$, $\beta^2 = Var(\ln\varepsilon)$.

The parameters m and β^2 can be related to the mean and variance of $\mathcal{E}$ by the following expression. [See Appendix 3.]

$$m = \ln \frac{\langle \varepsilon \rangle}{\sqrt{1 + \langle \varepsilon'^2 \rangle / \langle \varepsilon \rangle^2}} \quad , \quad \beta^2 = \ln \left(1 + \frac{\langle \varepsilon'^2 \rangle}{\langle \varepsilon \rangle^2}\right) \qquad (4.13)$$

Further postulated (Kolmogorov [10] and Obukov [11]) that the variance of $\mathcal{E}$ was locally related to the macro- and dissipation length scales of the turbulence by the formula

$$Var(\ln \varepsilon) = A + \mu_\varepsilon \ln \frac{L_0}{\eta} \qquad (4.15)$$

where A is a "constant" determined by the macro structure of the flow and μ_ε is an "universal" constant. L_0 is the macro-length scale determined from the integral of the two point correlation function and η is the dissipation length scale defined as

$$\eta = \left(\frac{\nu^3}{\langle \varepsilon \rangle}\right)^{\frac{1}{4}}$$

introducing (4.13) in (4.15) gives the following expression for the variance

$$\langle \varepsilon'^2 \rangle = \langle \varepsilon \rangle^2 \left[e^A \left(\frac{L_0}{\eta}\right)^{\mu_\varepsilon} - 1 \right] \qquad (4.16)$$

By asssuming that $\langle \varepsilon \rangle \simeq \alpha \left(\frac{\langle k \rangle^{\frac{3}{2}}}{L_0}\right)$ and defining a turbulent Reynolds number as $R_{e_T} = \langle k \rangle^2 / (\varepsilon \nu)$ (4.16) can be rewritten as

$$\langle \varepsilon'^2 \rangle = \langle \varepsilon \rangle^2 \left[e^A \alpha^{\mu_\varepsilon} R_{e_T}^{\frac{3}{4}\mu_\varepsilon} - 1 \right] \qquad (4.17)$$

Masiello [13] determined experimentally the constants A and μ_ε to be

$$A = -0.84 \quad , \quad \mu_\varepsilon = 0.47$$

From the work of Driscoll [14] the constant α is estimated as

$$\alpha = 0.5$$

4.1.3 The Transport Term

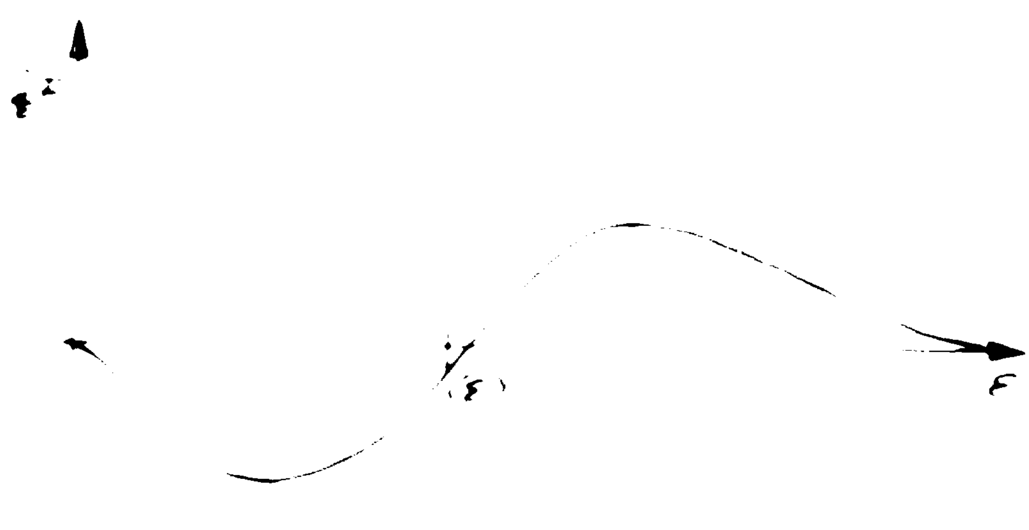
The term $-\hat{\sigma}_{E}^{\mathcal{T}}$ [illegible] expresses the [illegible] at $E = E^{(a)}$ due to the exchange of energy between the [illegible] zones. For large values of E [illegible] prevent the energy from becoming negative. The term [illegible] and must thus be negative for some small values of E [illegible] to the energy transport in the spectra [illegible] and [illegible] high wave numbers [illegible] dissipation rates [illegible] difference between the spectra [illegible] and the [illegible] the energy [illegible] spectra [illegible] the [illegible] between [illegible]

E [illegible]

[illegible]

$$-\hat{\sigma}_{E}^{\mathcal{T}} \;\hat{=}\; C_{M}\,(E - \langle E\rangle)\,\Gamma_{E} \qquad (4.8)$$

[illegible] $E < \langle E\rangle$ [illegible] $E > \langle E\rangle$ [illegible]



[illegible]

The modelled equation in the case of decaying isotropic turbulence then becomes

$$\partial_t(k^{(\alpha)}(\epsilon)P(\epsilon)) \simeq C_m(\epsilon-\langle\epsilon\rangle)P(\epsilon) - \epsilon P(\epsilon) \qquad (4.19)$$

where $P(\epsilon)$ can be calculated from (4.9) or (4.12)-(4.17).

4.4 Application of multiscale modelling to turbulent shear flows

The general equation for zone conditioned turbulent kinetic energy becomes, in the limit $\Delta\epsilon \to 0$

$$\partial_t(k^{(\alpha)}P) + \partial_\alpha(v_\alpha^{(\alpha)}k^{(\alpha)}P) = -\epsilon^{(\alpha)}P + \tau_{\alpha\beta}^{(\alpha)}P\,\partial_\beta v_\alpha^{(\alpha)}$$

$$+\,\partial_\beta(F'_{\alpha\alpha\beta}P) + \nu\partial^2_{\beta\beta}(k^{(\alpha)}P) - \partial_\epsilon F \qquad (4.20)$$

where $F'_{\alpha\alpha\beta}$ is the non-viscous part of $F_{\alpha\alpha\beta}$. When solving an equation for $v_\alpha^{(\alpha)}$ the only new terms to be modelled are the production term $\tau_{\alpha\beta}^{(\alpha)}P\partial_\beta v_\alpha^{(\alpha)}$ and the turbulent transport term $\partial_\beta(F'_{\alpha\alpha\beta}P)$. However, in this initial stage of model development the zone conditioned mean velocities will be assumed equal to the unconditional mean velocity.

$$v_\alpha^{(\alpha)} = \langle v_\alpha\rangle \qquad (4.21)$$

The zone conditioned shear stress $\tau_{\alpha\beta}^{(\alpha)}$ is assumed to be proportional to the unconditioned kinetic energy and the following closure model is suggested.

$$\tau_{\alpha\beta}^{(\alpha)} = C_D\frac{\langle k\rangle^2}{\langle\epsilon\rangle}\left(\partial_\beta\langle v_\alpha\rangle + \partial_\alpha\langle v_\beta\rangle\right) - \frac{2}{3}\delta_{\alpha\beta}\langle k\rangle \qquad (4.22)$$

$$C_D = 0.09$$

The transport term $F'_{\alpha\alpha\beta}P$ is modelled according to the turbulent viscosity concept

$$F'_{\alpha\alpha\beta}P \simeq C_D \frac{\langle k\rangle^2}{\langle\varepsilon\rangle}\partial_\beta(k^{(\alpha)}P) \qquad (4.23)$$

The modelled equation for zone conditional kinetic energy for the case of a two-dimensional turbulent shear flow then becomes

$$\partial_t(k^{(\alpha)}P) + \langle U\rangle\partial_x(k^{(\alpha)}P) + \langle V\rangle\partial_y(k^{(\alpha)}P) = \partial_y\left[\left(\nu + C_D\frac{\langle k\rangle^2}{\langle\varepsilon\rangle}\right)\partial_y(k^{(\alpha)}P)\right]$$

$$-\varepsilon^{(\alpha)}P + C_D P\frac{\langle k\rangle^2}{\langle\varepsilon\rangle}(\partial_y\langle U\rangle)^2 + C_M(\varepsilon^{(\alpha)} - \langle\varepsilon\rangle)P \qquad (4.24)$$

From this equation, two zones are constructed by integrating over ε from $0 \to \varepsilon_1$ and from $\varepsilon_1 \to \infty$. The kinetic energy in the two zones are defined as

$$\tilde{k}_1 = \int_0^{\varepsilon_1} d\varepsilon\, k^{(\alpha)}(\varepsilon)P(\varepsilon) \quad , \quad \tilde{k}_2 = \int_{\varepsilon_1}^{\infty} d\varepsilon\, k^{(\alpha)}(\varepsilon)P(\varepsilon)$$

The equation for $\tilde{k}_1$ and $\tilde{k}_2$ then reads

$$\partial_t\tilde{k}_1 + \langle U\rangle\partial_x\tilde{k}_1 + \langle V\rangle\partial_y\tilde{k}_1 = \partial_y\left[\left(\nu + C_D\frac{\langle k\rangle^2}{\langle\varepsilon\rangle}\right)\partial_y\tilde{k}_1\right]$$

$$-\tilde{\varepsilon}_1 + C_D\frac{\langle k\rangle^2}{\langle\varepsilon\rangle}(\partial_y\langle U\rangle)^2 F(\varepsilon_1) + C_M(\tilde{\varepsilon}_1 - \langle\varepsilon\rangle F(\varepsilon_1)) \qquad (4.25)$$

where $\tilde{\varepsilon}_1 = \int_0^{\varepsilon_1} d\varepsilon\,\varepsilon P(\varepsilon)$, $F(\varepsilon_1) = \int_0^{\varepsilon_1} d\varepsilon\, P(\varepsilon)$.

$$\partial_t\tilde{k}_2 + \langle U\rangle\partial_x\tilde{k}_2 + \langle V\rangle\partial_y\tilde{k}_2 = \partial_y\left[\left(\nu + C_D\frac{\langle k\rangle^2}{\langle\varepsilon\rangle}\right)\partial_y\tilde{k}_2\right]$$

$$-\tilde{\varepsilon}_2 + C_D\frac{\langle k\rangle^2}{\langle\varepsilon\rangle}(\partial_y\langle U\rangle)^2(1 - F(\varepsilon_1)) + C_M\left[\tilde{\varepsilon}_2 - \langle\varepsilon\rangle(1 - F(\varepsilon_1))\right]$$

$$(4.26)$$

where

$$\tilde{\varepsilon}_2 = \int_{\varepsilon_1}^{\infty} d\varepsilon \, \varepsilon \, P(\varepsilon)$$

In cases where the mean dissipation $\langle\varepsilon\rangle$ has a large spatially variation it may not be possible to find a representative value of ε_1 for the whole flow field. Then it is better to express ε_1 as a fraction of $\langle\varepsilon\rangle$ and transform the equations (4.25) and (4.26) accordingly. This transformation is shown in Appendix 4.

5. Results

5.1 Decaying isotropic turbulence

In order to solve eq. (4.24) the following dimensionless variables are introduced

$$\check{t} = \frac{t}{\tau_o} \quad , \quad \check{P} = P \cdot \langle\varepsilon\rangle_o \quad , \quad \check{\varepsilon} = \frac{\varepsilon}{\langle\varepsilon\rangle_o} \quad , \quad \langle\check{\varepsilon}\rangle = \frac{\langle\varepsilon\rangle}{\langle\varepsilon\rangle_o}$$

$$\check{k}^{(k)} = \frac{k^{(k)}}{\langle k\rangle_o} \quad , \quad \langle\check{\varepsilon}'^2\rangle = \frac{\langle\varepsilon'^2\rangle}{\langle\varepsilon'^2\rangle_o}$$

where $\tau_o = \frac{\langle k\rangle_o}{\langle\varepsilon\rangle_o}$ and the subscript o indicates the value of the variable when $t = 0$.

Equation (4.24) transforms then into

$$\partial_{\check{t}} (\check{k}^{(k)} \check{P}) = - \check{\varepsilon} \check{P} - C_1 \check{P} (\check{\varepsilon} - \langle\check{\varepsilon}\rangle) \tag{5.1}$$

The initial Pdf is taken as log-normal and the distribution of $\check{k}^{(k)}$ as a function of $\check{\varepsilon}$ is assumed proportional to $\check{P}(\check{\varepsilon})$.

$$\check{k}^{(k)} = \frac{\check{P}(\check{\varepsilon})}{\int_0^{\infty} d\check{\varepsilon} \, \check{P}^2(\check{\varepsilon})}$$

which satisfies the requirement

$$\int_0^{\infty} d\check{\varepsilon} \, \check{k}^{(k)} \check{P} = 1$$

Equation (5.1) is solved by using both a prescribed pdf and by solving the pdf-transport equation. For the results shown in figs. 5-8 the pdf is prescribed as log-normal and Kolmogorov's expression for the variance is used. The mean dissipation, which is a parameter of the pdf, is calculated from the standard $\langle k \rangle - \langle \varepsilon \rangle$ model.

Fig. 5 shows the distribution of $\check{k}^{(k)}$ as a function of $\check{\varepsilon}$ for three different times $\check{t}$ = 0, 0.21, 0.42. Fig. 6 shows the "spectrum" $\check{k}^{(k)}/\check{P}$, fig. 7 shows the interzonal transport of energy F and fig. 8 shows the evolution of the pdf. The results are in qualitative agreement with what one would expect, but due to the complete lack of experimental data it is not possible to draw any conclusion about the quantitative behavior.

Figs. 9-12 show the corresponding results when eq. (5.1) is solved together with the transport equation for $\check{P}(\check{\varepsilon})$. The constant A in the pdf equation is calculated from eq. (8) in Appendix 2 and C_1 is put equal to $C_1 = 0.1$. The results from this calculation are nearly identical witn the results shown in figs. 5-8.

5.2 Plane jet

The results from the calculation of a plane jet are shown in figs. 13-16. The value of the discriminating scalar ε_1 that defines the two zones was set equal to $\varepsilon = 10^{+3}$ m^2/s^3. The exit velocity and the nozzle width were $U_e = 35$ m/s and $D = 3 \times 10^{-3} m$, respectively. Figures 13a-c show a typical development of the zonal kinetic energies $\tilde{k}_1$ and $\tilde{k}_2$. Figure 13a shows that for x/D = 10 the main part of the kinetic energy is contained in zone 2 ($\varepsilon > \varepsilon_1$). Further downstream the dissipation rate decreases and more and more of the energy is contained in zone 1 ($\varepsilon < \varepsilon_1$). For X/D = 60 fig. 13c zone 2 is drained out and all the energy is contained in zone 1. At this station the two zone model acts as a one-zone model. It is possible, however, to introduce a non-constant

value of the discriminating scalar such that both zones always contain a non-neglibile amount of energy as shown in Appendix 4. Figures 14a-c show the development of the probability for $\varepsilon < \varepsilon_1$ calculated as the integral from $\varepsilon = 0$ to $\varepsilon = \varepsilon_1$ of the log-normal distribution. Due to the decrease in $\langle \varepsilon \rangle$ this probability increases downstream. In fig. 15 the interzonal energy transfer is shown. This energy transfer is negative over the whole cross-section (x/D = 60) which means that energy is transferred from zone 1 to zone 2, but it is not strong enough to keep up with decrease of $\tilde{k}_2$ due to the decrease in the dissipation rate. Figure 16 shows the mean and the standard deviation of the dissipation rate. The standard deviation is everywhere larger than the mean value as calculated from the refined similarity hypothesis (eq. (4.17)).

Conclusions

A new multi-scale (multi-zone) concept for turbulent flows has been developed, and exact equations valid inside the different zones are derived and some simple initial modelling assumptions have been constructed in order to close the system of equations. The model is applied to calculate decaying isotropic turbulence and a plane jet. The results are qualitatively in agreement with expectaions, but due to the complete lack of experimental data it is not possible to draw any further conclusions. More work is probably necessary in order to refine the modelling and to extend the multi-scale concept to second order level.

Acknowledgement

This work was supported by AFOSR Grant 84-0[illegible].

APPENDIX 1

PDF equation for $\tilde{\varepsilon}$

Let $\tilde{\varepsilon}$ be given as $\tilde{\varepsilon} = \nu \partial_s u_k \partial_s u_k$. An exact transport equation for $\tilde{\varepsilon}$ can be derived by taking the derivative of the instantaneous momentum equation

$$\partial_t u_k + u_s \partial_s u_k = -\frac{1}{\rho} \partial_k p + \nu \partial^2_{ss} u_k$$

w.r.t. x_j and multiply the result with $\partial_j u_k$.

The result is

$$\partial_t \tilde{\varepsilon} + u_s \partial_s \tilde{\varepsilon} = \nu \partial^2_{ss} \tilde{\varepsilon} - 2 \frac{\nu}{\rho} \partial_s u_k \partial^2_{sk} p - 2\nu \partial^2_{sw} u_k \partial^2_{sw} u_k$$
$$- 2\nu \partial_j u_k \partial_s u_k \partial_s u_j \qquad (1)$$

where the pressure can be related to the velocity field by the Poisson integral.

$$p(\underline{x}) = \frac{1}{4\pi} \iiint_{\mathcal{D}} \frac{dy}{|\underline{x} - \underline{y}|} \frac{\partial v_r}{\partial y_s} \frac{\partial v_s}{\partial y_r} + \{\text{Boundary term}\}$$

Denoting by ε and v_k the probability space variables corresponding to $\tilde{\varepsilon}$ and u_k , respectively and defining a fine-grained joint Pdf for velocity and dissipation as

$$\hat{P}(v_k, \varepsilon, \underline{x}, t) = \delta(\tilde{\varepsilon} - \varepsilon) \prod_{k=1}^{3} \delta(u_k - v_k)$$

the equation for $P(v_k, \varepsilon, \underline{x}, t) = \langle \hat{P}(v_k, \varepsilon, \underline{x}, t) \rangle$ is easily obtained by using standard operations with the δ-function.

$$\partial_t P + v_\beta \partial_\beta P = \nu \partial^2_{\beta\beta} P - \nu \frac{\partial^2}{\partial v_\alpha \partial v_\gamma} \langle \partial_\beta v_\alpha \partial_\beta v_\gamma \hat{P} \rangle$$

$$- \nu \frac{\partial^2}{\partial \varepsilon^2} \langle \partial_\beta \tilde{\varepsilon} \partial_\beta \tilde{\varepsilon} \hat{P} \rangle - 2\nu \frac{\partial^2}{\partial v_\alpha \partial \varepsilon} \langle \partial_\beta v_\alpha \partial_\beta \tilde{\varepsilon} \hat{P} \rangle$$

$$- \partial_\alpha \left[\frac{1}{4\pi} \iiint_{D} \frac{dy}{|\underline{x} - \underline{y}|} \frac{\partial v_\gamma}{\partial y_\beta} \frac{\partial v_\beta}{\partial y_\gamma} \frac{\partial \hat{P}}{\partial v_\alpha} \right] + 2\nu \frac{\partial}{\partial \varepsilon} \langle \left(\frac{1}{\rho} \partial_\beta v_\alpha \partial^2_{\alpha\beta} p \right.$$

$$\left. + \nu \partial^2_{\beta\gamma} v_\alpha \partial^2_{\beta\gamma} v_\alpha + \nu \partial_\gamma v_\alpha \partial_\beta v_\alpha \partial_\beta v_\gamma \right) \hat{P} \rangle \qquad (2)$$

By integrating this equation over the velocity space the equation for the marginal Pdf of ε, $P(\varepsilon)$ is obtained as

$$\partial_t P + \partial_\beta (\langle v_\beta | \varepsilon \rangle P) = \nu \partial^2_{\beta\beta} P - \nu \frac{\partial^2}{\partial \varepsilon^2} \langle \partial_\beta \tilde{\varepsilon} \partial_\beta \tilde{\varepsilon} \bar{P} \rangle$$

$$+ 2\nu \frac{\partial}{\partial \varepsilon} \langle \left(\frac{1}{\rho} \partial^2_{\alpha\beta} p \, \partial_\beta v_\alpha + \nu \partial^2_{\beta\gamma} v_\alpha \partial^2_{\beta\gamma} v_\alpha + \nu \partial_\gamma v_\alpha \partial_\beta v_\alpha \partial_\beta v_\gamma \right) \bar{P} \rangle$$

where

$$\bar{P} \equiv \delta(\tilde{\varepsilon}(\underline{x}, t) - \varepsilon)$$

APPENDIX 2

Analytical solution for the variance of ε in the case of isotropic turbulence

Consider the following form of the equation for the variance of ε in the case of isotropic turbulence.

$$\frac{d\langle\varepsilon'^2\rangle}{dt} = -\frac{C_3}{\tau}\langle\varepsilon'^2\rangle \qquad (1)$$

where $\tau = \frac{\langle k\rangle}{\langle\varepsilon\rangle}$ and where $\langle k\rangle$ and $\langle\varepsilon\rangle$ can be found from the $\langle k\rangle - \langle\varepsilon\rangle$ model as

$$\frac{\langle k\rangle}{\langle k\rangle_0} = \left[1 + (C_2 - 1)\frac{t}{\tau_0}\right]^{-\frac{1}{C_2 - 1}} \qquad (2)$$

$$\frac{\langle\varepsilon\rangle}{\langle\varepsilon\rangle_0} = \left[1 + (C_2 - 1)\frac{t}{\tau_0}\right]^{-\frac{C_2}{C_2 - 1}} \qquad (3)$$

where $\langle k\rangle_0 = \langle k\rangle(t=0)$, $\langle\varepsilon\rangle_0 = \langle\varepsilon\rangle(t=0)$

and $\tau_0 = \frac{\langle k\rangle_0}{\langle\varepsilon\rangle_0}$.

The solution of (1) is then

$$\frac{\langle\varepsilon'^2\rangle}{\langle\varepsilon'^2\rangle_0} = \left[1 + (C_2 - 1)\frac{t}{\tau_0}\right]^{-\frac{C_3}{C_2 - 1}} \qquad (4)$$

From (2) and (3) it can be seen that the turbulent Reynolds number $R_t = \langle k\rangle^2/\nu\langle\varepsilon\rangle$ decreases with time when $1.0 < C_2 < 2.0$. The commonly used value is $C_2 = 1.9$ which is consistent with a decrease of Re. The requirement that the ratio $\langle\varepsilon'^2\rangle/\langle\varepsilon\rangle$ is increasing with increasing Re gives the following condition on C_3

$$C_3 > 2C_2 \tag{5}$$

leading to the following condition on A from (4.11)

$$A > 0 \tag{6}$$

The constant C_3 can also be related to the constant μ in the Kolmogorov expression for the variance of ε by the requirement that the ratio $\langle \varepsilon'^2 \rangle / \langle \varepsilon \rangle$ calculated from (3) and (4) should have the same time dependence as (4.17). The result is

$$C_3 = 2C_2 \left(1 + \frac{3}{4}\mu \frac{2 - C_2}{2C_2}\right) \tag{7}$$

The constant A in (4.11) can then be given by

$$A = \frac{9}{4}\mu (2 - C_2) \tag{8}$$

The constant μ can further be related to the fractal dimension of intermittent turbulence.

APPENDIX 3

Integrals and moments of the log-normal distribution

Let ε be a stochastic, log-normally distributed variable. The logarithm of ε is then normal distributed and the Pdf of $\ln \varepsilon$ is given by

$$P_{\ln\varepsilon}(\ln\varepsilon) = \frac{1}{\beta\varepsilon\sqrt{2\pi}} \exp\left\{-\left(\frac{\ln\varepsilon - m}{\beta\sqrt{2}}\right)^2\right\} \qquad (1)$$

where $m = \langle \ln\varepsilon \rangle$, $\beta^2 = \langle (\ln\varepsilon - \langle \ln\varepsilon \rangle)^2 \rangle$.

The Pdf of ε, $P_\varepsilon(\varepsilon)$ can then be determined as

$$P_\varepsilon(\varepsilon) = P_{\ln\varepsilon}(\varepsilon) \Big/ \left|\frac{d\ln\varepsilon}{d\varepsilon}\right| = \frac{1}{\varepsilon} P_{\ln\varepsilon}(\varepsilon) \equiv P(\varepsilon)$$

which gives for $P(\varepsilon)$

$$P(\varepsilon) = \frac{1}{\beta\varepsilon\sqrt{2\pi}} \exp\left\{-\frac{1}{2\beta^2}(\ln\varepsilon - m)^2\right\} \qquad (2)$$

Consider the integral $I_{\varepsilon_1}^{p}$

$$I_{\varepsilon_1}^{p} \equiv \int_0^{\varepsilon_1} d\varepsilon \, \varepsilon^p P(\varepsilon)$$

Introducing (2) gives

$$I_{\varepsilon_1}^{p} = \frac{1}{\beta\sqrt{2\pi}} \int_0^{\varepsilon_1} d\varepsilon \, \varepsilon^{p-1} \exp\left\{-\frac{1}{2\beta^2}(\ln\varepsilon - m)^2\right\}$$

Introducing a new variable $U(\varepsilon)$ defined by

$$U(\varepsilon) = \frac{\ln \varepsilon - m}{\beta\sqrt{2}} \quad , \quad dU = \frac{d\varepsilon}{\beta\varepsilon\sqrt{2}}$$

gives the following expression for $I^{p}_{\varepsilon_1}$

$$I^{p}_{\varepsilon_1} = \frac{e^{pm}}{\sqrt{\pi}}\int_{-\infty}^{U(\varepsilon_1)} dU \exp(\sqrt{2}\beta p U - U^2)$$

This integral can be further transformed into (see Ref. [15], p. 303)

$$I^{p}_{\varepsilon_1} = \frac{1}{2}\exp(pm + \tfrac{1}{2}\beta^2 p^2)\left[\operatorname{erf}\left(U(\varepsilon_1) - \frac{\beta p}{\sqrt{2}}\right) + 1\right] \quad (3)$$

where $\operatorname{erf}(\cdot)$ is the error function defined as

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x dt\, e^{-t^2}$$

The moments of ε can now be calculated as

$$\langle \varepsilon^p \rangle = I^{p}_{\infty} = e^{pm + \frac{1}{2}p^2\beta^2} \quad (4)$$

By combining the first and second order moments the parameters m and β^2 can be expressed as

$$\beta^2 = \ln\left(1 + \frac{\langle \varepsilon'^2 \rangle}{\langle \varepsilon \rangle^2}\right) \quad , \quad m = \ln \frac{\langle \varepsilon \rangle}{\sqrt{1 + \frac{\langle \varepsilon'^2 \rangle}{\langle \varepsilon \rangle^2}}} \quad (5)$$

where $\langle \varepsilon'^2 \rangle = \langle (\varepsilon - \langle \varepsilon \rangle)^2 \rangle$.

The integral $I^{P}_{\varepsilon_1}$ must be calculated numerically, this can easily be done by one of the approximations for the error-function given in Ref. 15. In this work the following approximation is used.

$$erf(x) = 1 - e^{-x^2} \sum_{k=1}^{5} a_k t^k \qquad , x \geq 0$$

where $t = \dfrac{1}{1+px}$, and the constants are

P = 0.3275911 , a_1 = 0.254829592 , a_2 = − 0.284496736

a_3 = 1.421413741 , a_4 = −1.453152027 , a_5 = 1.061405429

For $x<0$ the symmetry properties of $erf(x)$ can be used

$$erf(x) = -erf(-x)$$

APPENDIX 4

Transformation of the multiscale equation

By defining $A \equiv k^{(\kappa)} P(\varepsilon)$ the modelled equation for zone conditioned kinetic energy can be written as

$$\partial_t A + \langle v_\kappa \rangle \partial_\kappa A = \partial_\kappa (\mu_t \partial_\kappa A) + S_A \qquad (1)$$

where S_A is the collection of production/dissipation and interzonal transport terms.

A new discriminating scalar $\hat{\varepsilon}$ defined as

$$\hat{\varepsilon} = \frac{\varepsilon}{\langle \varepsilon \rangle}$$

and a new dependent variable $\hat{A}$ defined as

$$\hat{A} = k^{(\kappa)} P_{\hat{\varepsilon}}(\hat{\varepsilon}) \;, \quad P_{\hat{\varepsilon}}(\hat{\varepsilon}) = \langle \varepsilon \rangle P_{\varepsilon}(\hat{\varepsilon})$$

are introduced such that

$$\int_0^\infty d\hat{\varepsilon}\, \hat{A}(\hat{\varepsilon}) = \langle k \rangle$$

Then follows $\hat{A} = \langle \varepsilon \rangle A$ and the equation for $\hat{A}$ becomes

$$\partial_t \hat{A} + \langle v_\kappa \rangle \partial_\kappa \hat{A} = \partial_\kappa (\mu_t \partial_\kappa \hat{A}) - 2\mu_t \partial_\kappa \langle \varepsilon \rangle \partial_\kappa \left(\frac{\hat{A}}{\langle \varepsilon \rangle} \right)$$

$$+ S_A + S_{\langle \varepsilon \rangle} \frac{\hat{A}}{\langle \varepsilon \rangle} \qquad (2)$$

where $S_{\langle \varepsilon \rangle}$ is the source term in the $\langle \varepsilon \rangle$-equation.

Equation [illegible] is now transformed to a new coordinate system by the transformation

$$(x_\alpha, t, \mathcal{E}) \Longleftrightarrow (\hat{x}_\alpha, \hat{t}, \hat{\mathcal{E}})$$

where $\hat{x}_\alpha = x_\alpha$, $\hat{t} = t$, $\hat{\mathcal{E}} = \frac{\mathcal{E}}{\langle \mathcal{E} \rangle}$

By using the following transformation rules

$$\partial_\alpha = \partial_{\hat{\alpha}} - \frac{\hat{\mathcal{E}}}{\langle \mathcal{E} \rangle} \partial_\alpha \langle \mathcal{E} \rangle \partial_{\hat{\mathcal{E}}}$$

$$\partial_t = \partial_{\hat{t}} - \frac{\hat{\mathcal{E}}}{\langle \mathcal{E} \rangle} \partial_t \langle \mathcal{E} \rangle \partial_{\hat{\mathcal{E}}}$$

the following transformed equation results

$$\partial_t \hat{A} + \langle U_\alpha \rangle \partial_\alpha \hat{A} = \partial_\alpha (\mu_t \partial_\alpha \hat{A}) - 2 \frac{\mu_t}{\langle \mathcal{E} \rangle} \partial_\alpha \langle \mathcal{E} \rangle \partial^2_{\alpha \hat{\mathcal{E}}} (\hat{A} \hat{\mathcal{E}})$$

$$+ \frac{\mu_t}{\langle \mathcal{E} \rangle^2} \partial_\alpha \langle \mathcal{E} \rangle \partial_\alpha \langle \mathcal{E} \rangle \partial^2_{\hat{\mathcal{E}} \hat{\mathcal{E}}} (\hat{\mathcal{E}}^2 \hat{A}) + \frac{1}{\langle \mathcal{E} \rangle} S_{\langle \mathcal{E} \rangle} \partial_{\hat{\mathcal{E}}} (\hat{\mathcal{E}} \hat{A}) + S_A \qquad (3)$$

Two zones are now defined, one for $\hat{\mathcal{E}} \leq 1$ and one for $\hat{\mathcal{E}} \geq 1$. It will now be shown how the equation for the kinetic energy in zone 1, $\tilde{k}_1$ can be obtained. The equation for $\tilde{k}_2$ follows similarly.

Integrating [illegible] over the interval $0 \leq \hat{\mathcal{E}} \leq 1$ gives

$$\partial_t \tilde{k}_1 + \langle U_\alpha \rangle \partial_\alpha \tilde{k}_1 = \partial_\alpha (\mu_t \partial_\alpha \tilde{k}_1) - 2 \frac{\mu_t}{\langle \mathcal{E} \rangle} \partial_\alpha \langle \mathcal{E} \rangle \partial_\alpha [\hat{A}]_{\hat{\mathcal{E}}=1}$$

$$+ \frac{\mu_t}{\langle \mathcal{E} \rangle^2} \partial_\alpha \langle \mathcal{E} \rangle \partial_\alpha \langle \mathcal{E} \rangle \left[2\hat{A} + \partial_{\hat{\mathcal{E}}} \hat{A} \right]_{\hat{\mathcal{E}}=1} +$$

$$+ S_{\langle \mathcal{E} \rangle} \frac{1}{\langle \mathcal{E} \rangle} [\hat{A}]_{\hat{\mathcal{E}}=1} + \int_0^1 d\hat{\mathcal{E}} \, S_A \qquad (4)$$

The terms $[\hat{A}]_{\hat{\mathcal{E}}=1}$ and $[\partial_{\hat{\mathcal{E}}} \hat{A}]_{\hat{\mathcal{E}}=1}$ are unknown.

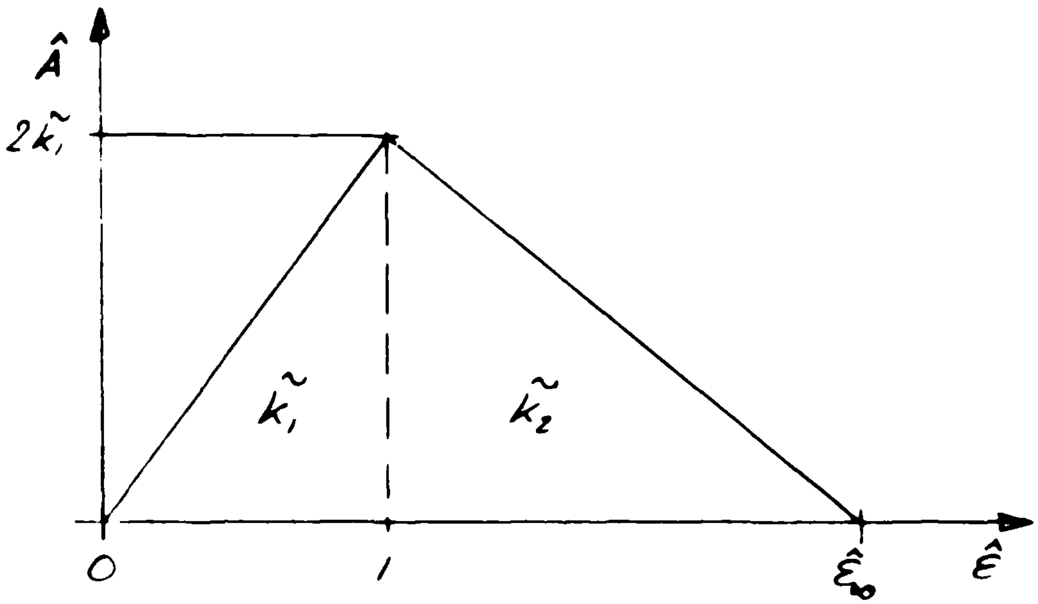
If an infinite number of zones is used the form of $\hat{A}(\hat{\varepsilon})$ is known, but in the cas of a finite number of zones $\hat{A}(\hat{\varepsilon})$ must be approximated.

When only two zones are used we suggest the following approximation for $\hat{A}(\hat{\varepsilon})$.

$$\hat{A} = \begin{cases} 2\tilde{k}_1 \hat{\varepsilon} & \text{for } 0 \le \hat{\varepsilon} \le 1 \\ 2\tilde{k}_1 \left(1 - \dfrac{\hat{\varepsilon}}{\hat{\varepsilon}_\infty}\right) & \text{for } 1 \le \hat{\varepsilon} \le \hat{\varepsilon}_\infty \\ 0 & \text{for } \hat{\varepsilon} > \hat{\varepsilon}_\infty \end{cases} \tag{5}$$

where $\hat{\varepsilon}_\infty$ is given by $\hat{\varepsilon}_\infty = 1 + \dfrac{\tilde{k}_2}{\tilde{k}_1}$.

This approximation is illustrated in fig. 1.



which gives $[\hat{A}]_{\hat{\varepsilon}=1} = 2\tilde{k}_1$ and $[\partial_{\hat{\varepsilon}} \hat{A}]_{\hat{\varepsilon}=1}$ can be approximated as

$$[\partial_{\hat{\varepsilon}} \hat{A}]_{\hat{\varepsilon}=1} \cong \frac{1}{2}\left[\partial_{\hat{\varepsilon}} \hat{A}\big|_{\hat{\varepsilon}=1^+} + \partial_{\hat{\varepsilon}} \hat{A}\big|_{\hat{\varepsilon}=1^-}\right] = \tilde{k}_1\left(1 - \frac{\tilde{k}_1}{\tilde{k}_2}\right)$$

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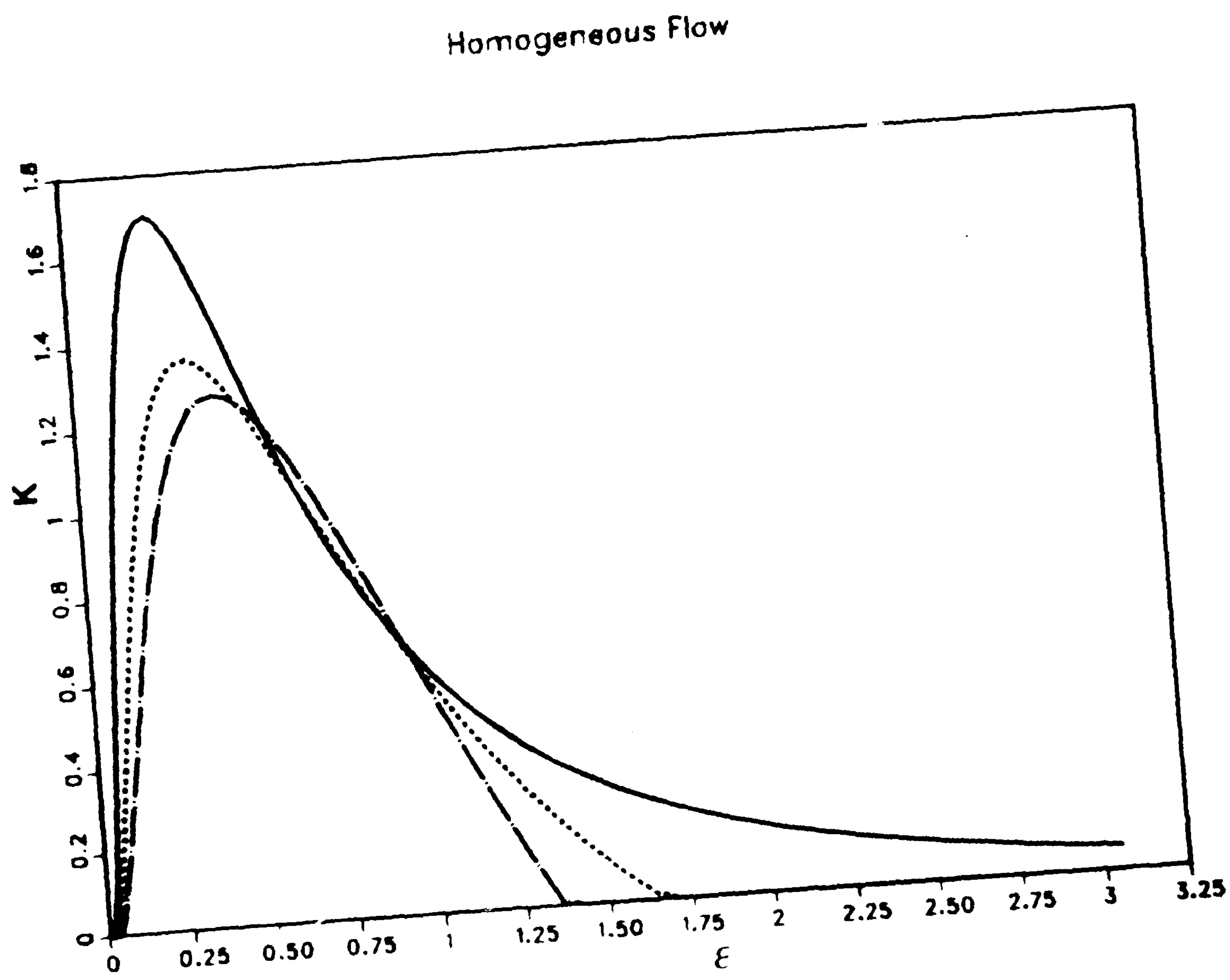

Fig. 5 Distribution of $k^{(\kappa)}$ for log normal $P(\varepsilon)$.

Homogeneous Flow

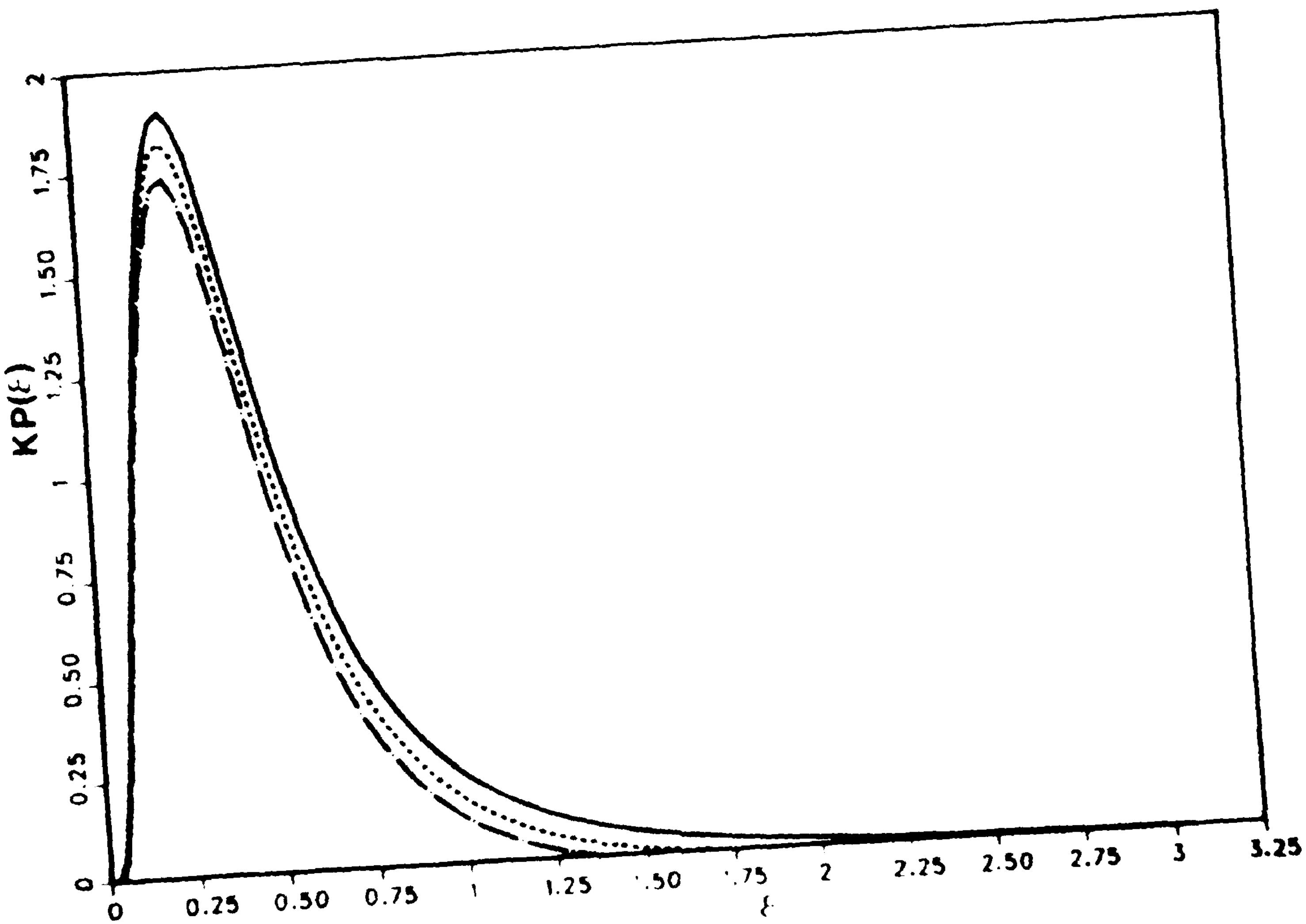

Fig. 6 Distribution of KP for log normal $P(\xi)$

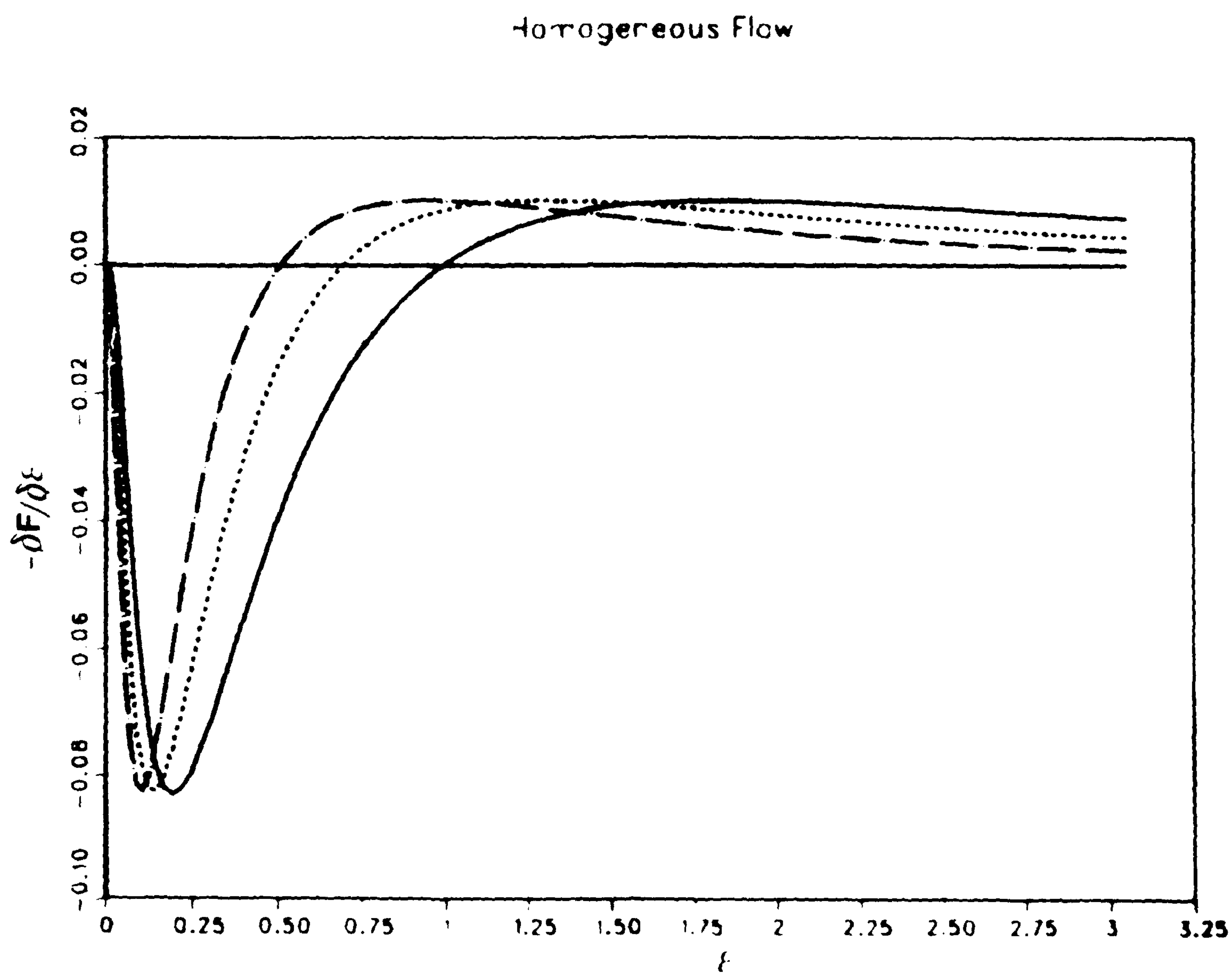

Fig. 7 The energy transfer term for log normal $P(\varepsilon)$

Homogeneous Flow

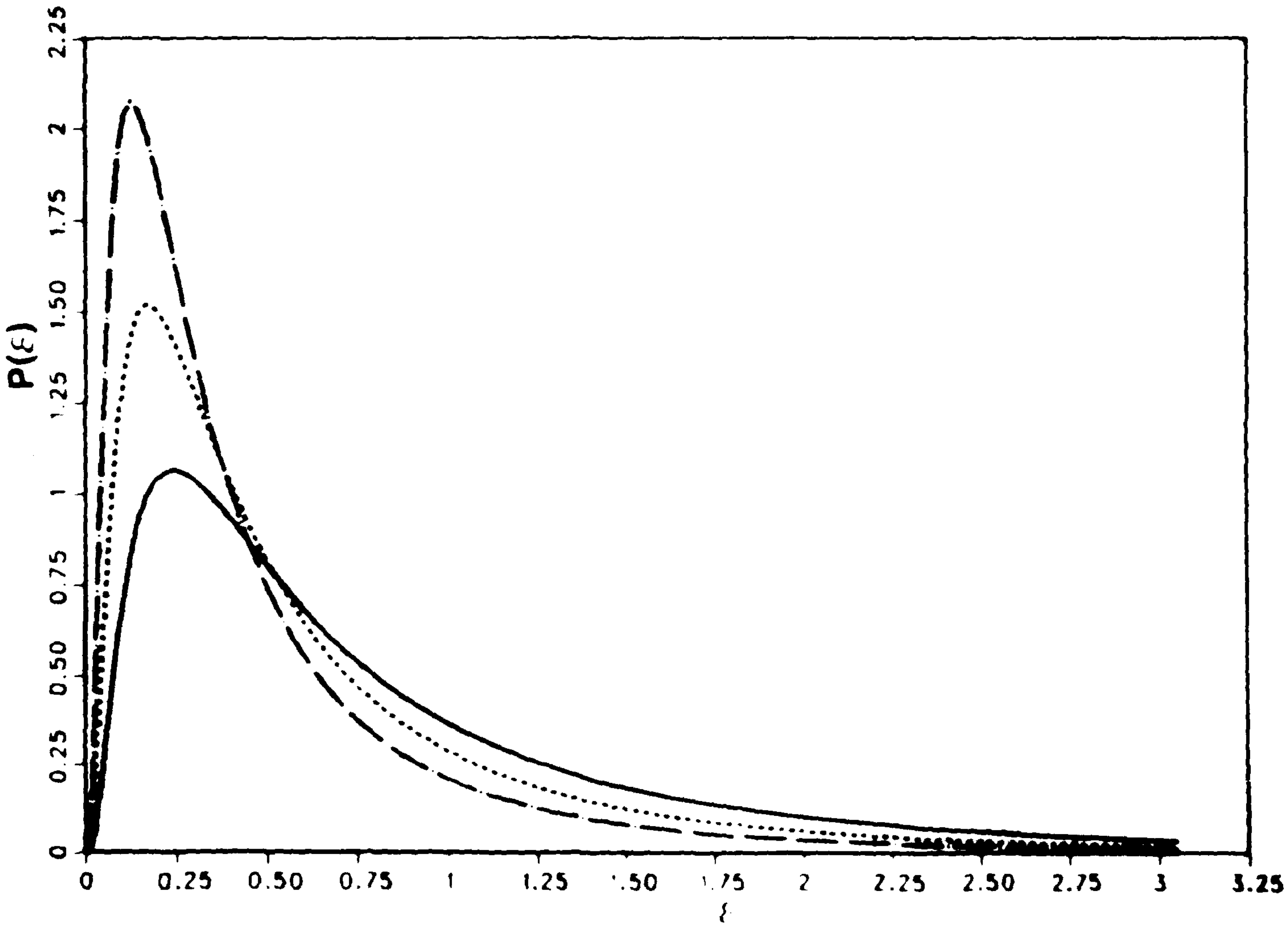

Fig. 8 Evolution of the log-normal $P(\varepsilon)$.

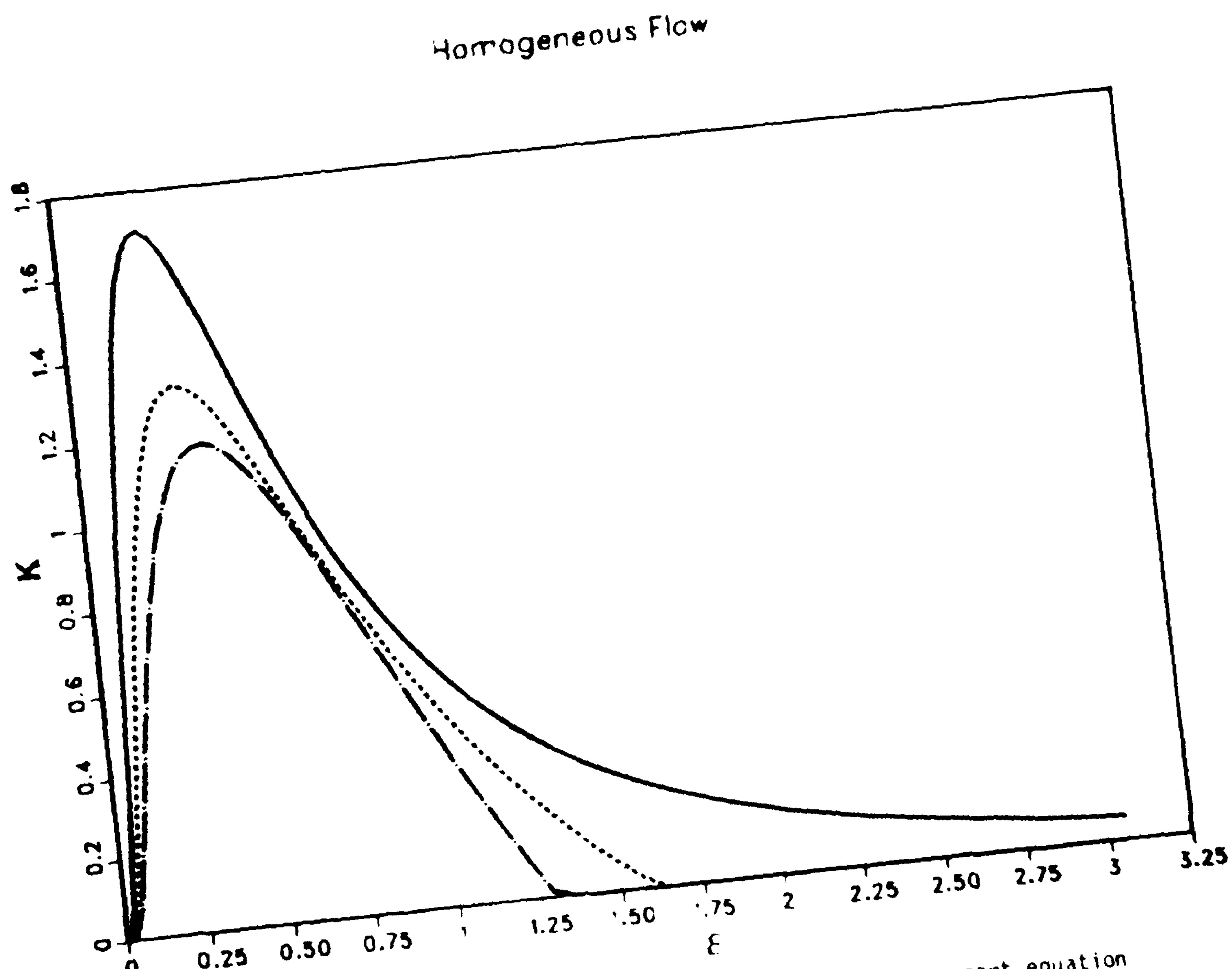

Fig. 9 Distribution of $k^{(\varepsilon)}$, $P(\varepsilon)$ calculated from transport equation

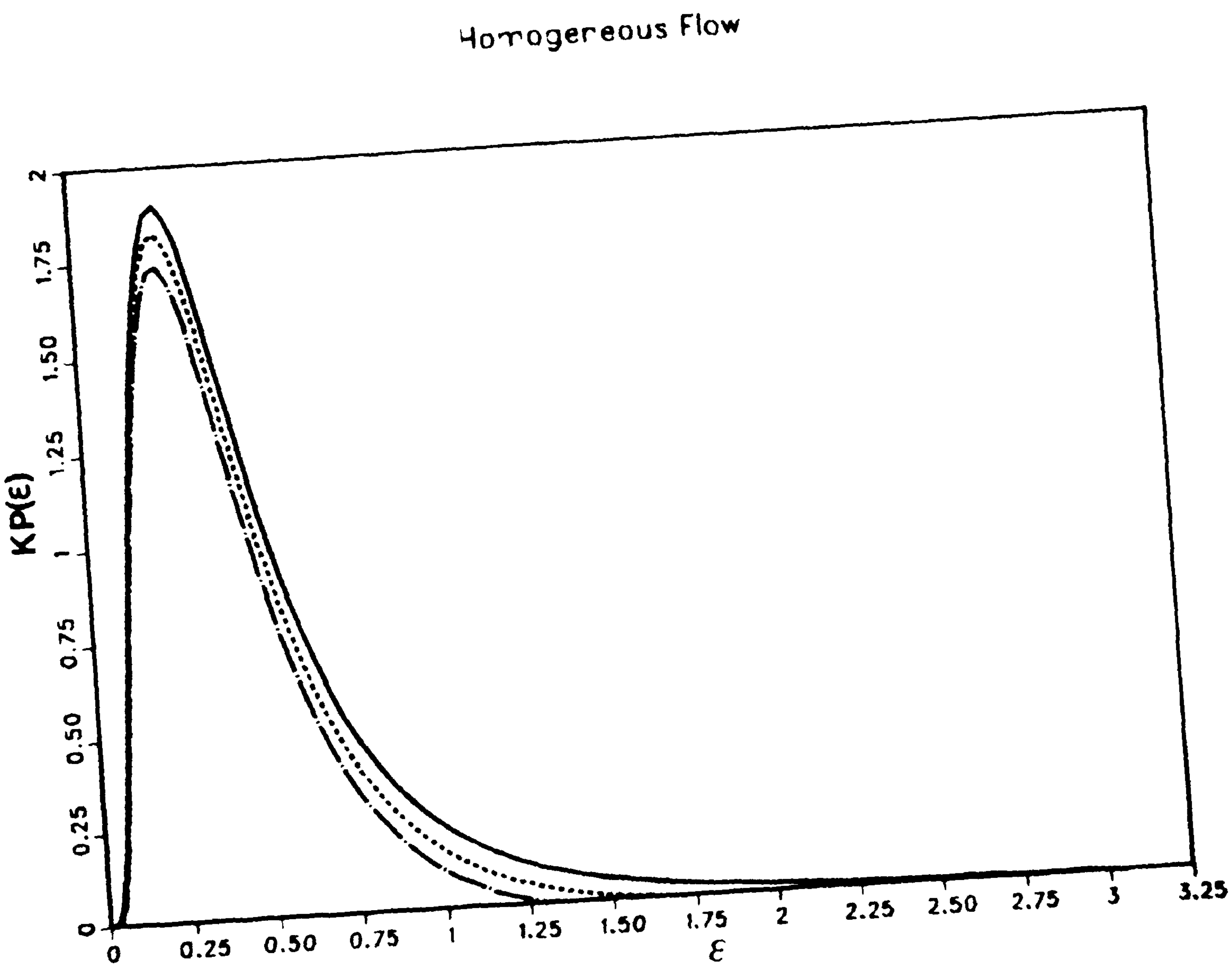

Fig. 10 Distribution of $k^{(n)}(\varepsilon)\,P(\varepsilon)$. $P(\varepsilon)$ calculated from transport equation.

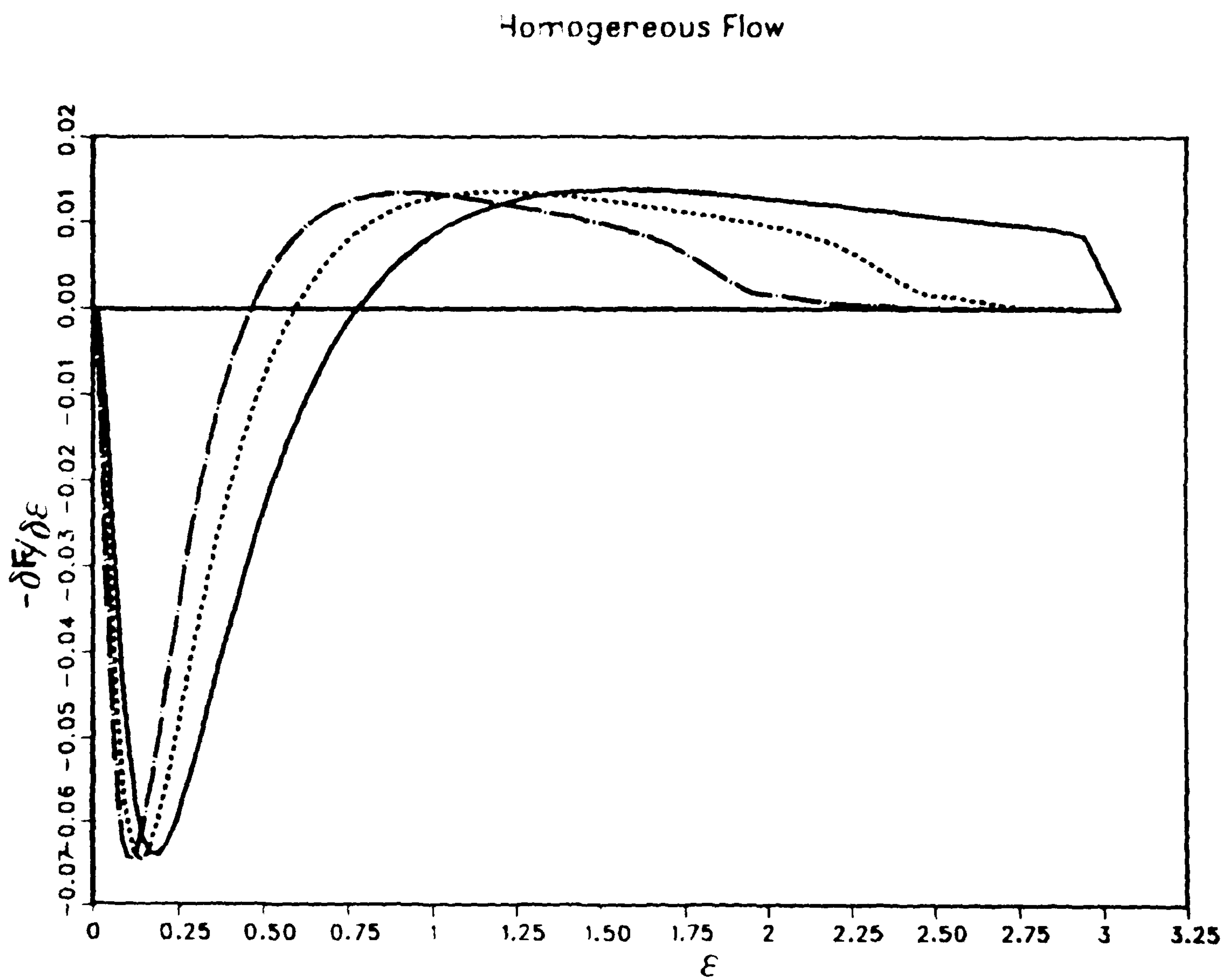

Fig. 11 The energy transfer term. $P(\varepsilon)$ calculated from transport equation

Homogeneous Flow

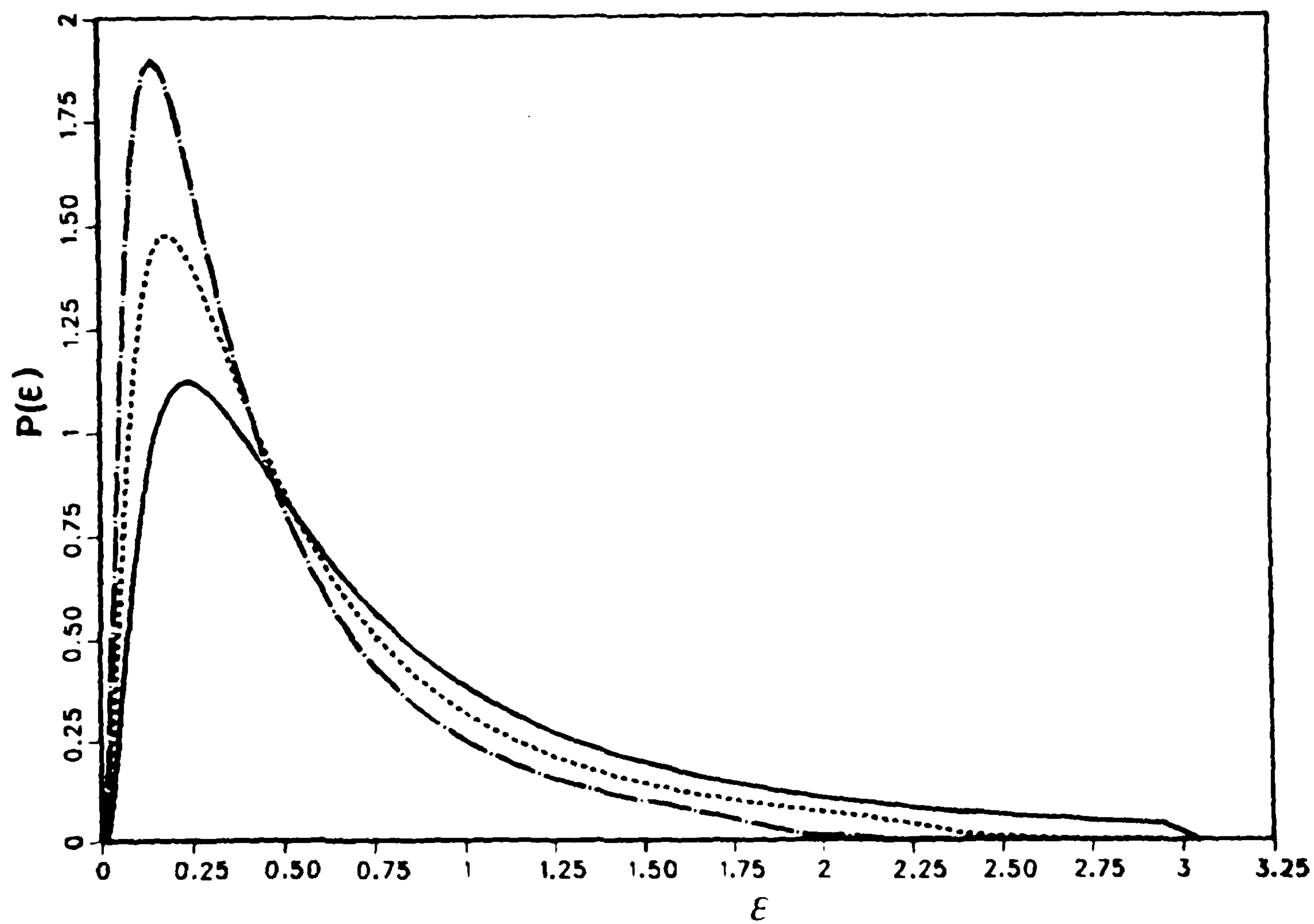

Fig. 12 Evolution of $P(\varepsilon)$ as calculated from the transport equation.

Plane jet

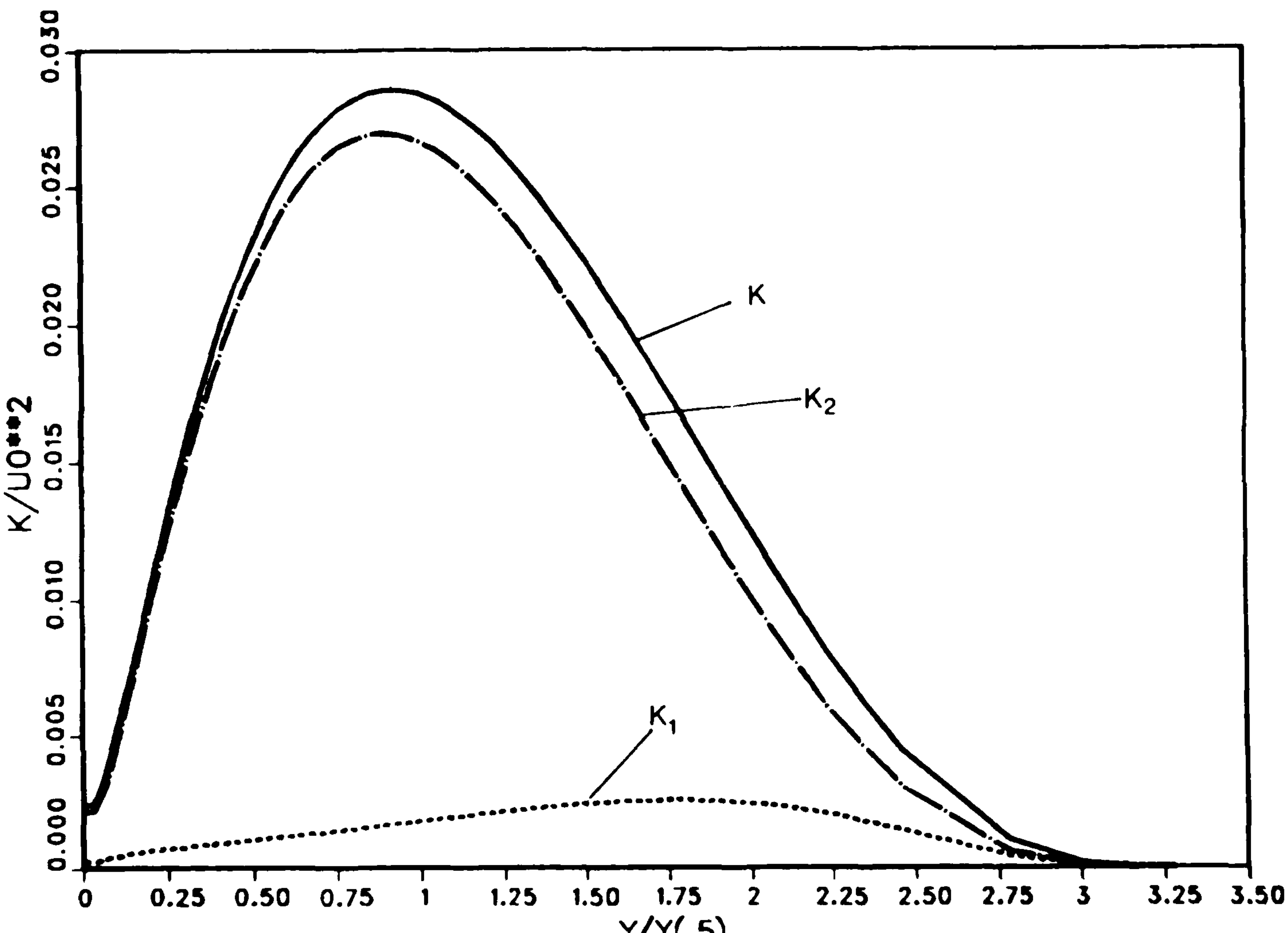

Fig.13 a

Plane jet

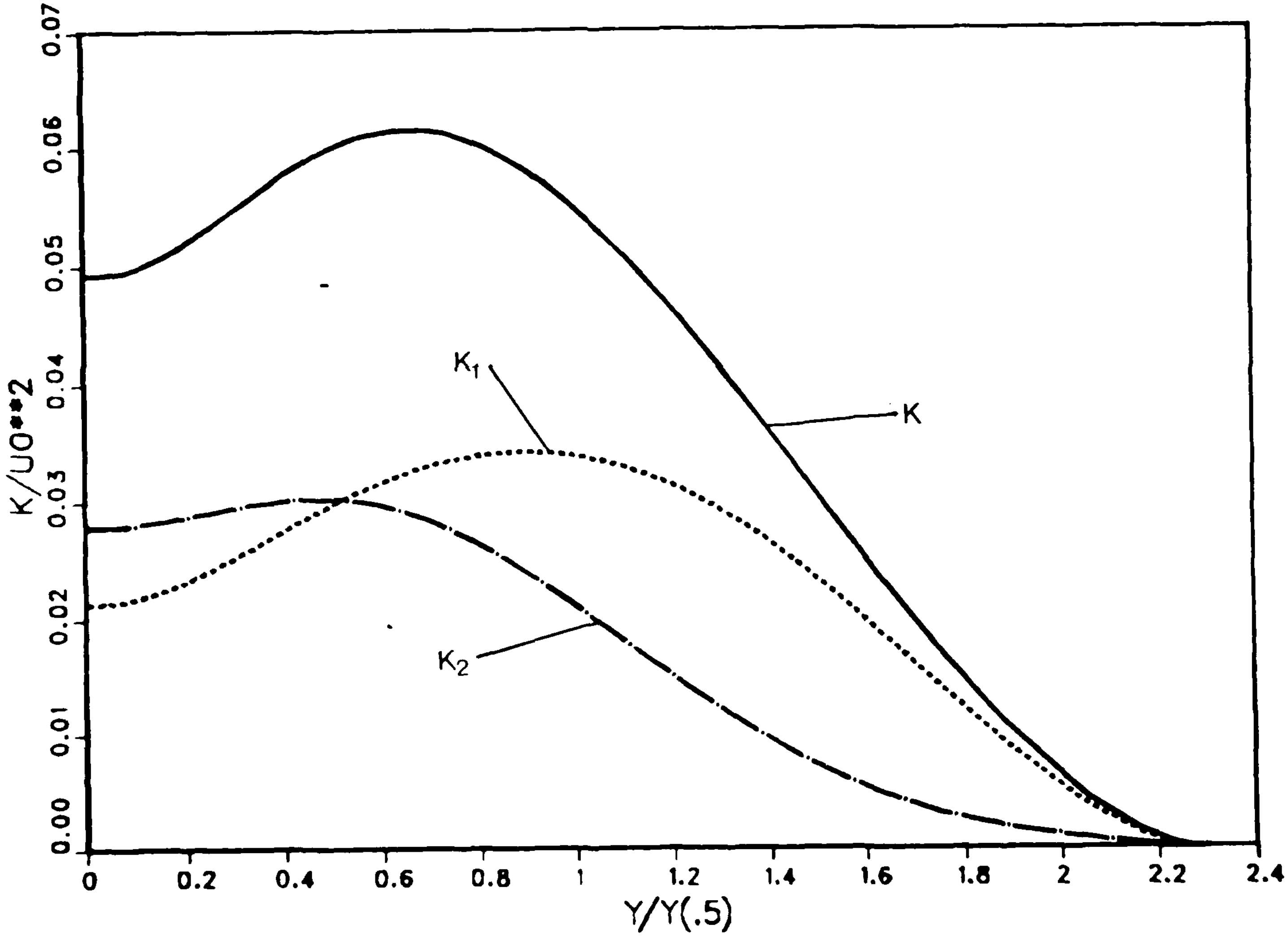

Fig. 13 b

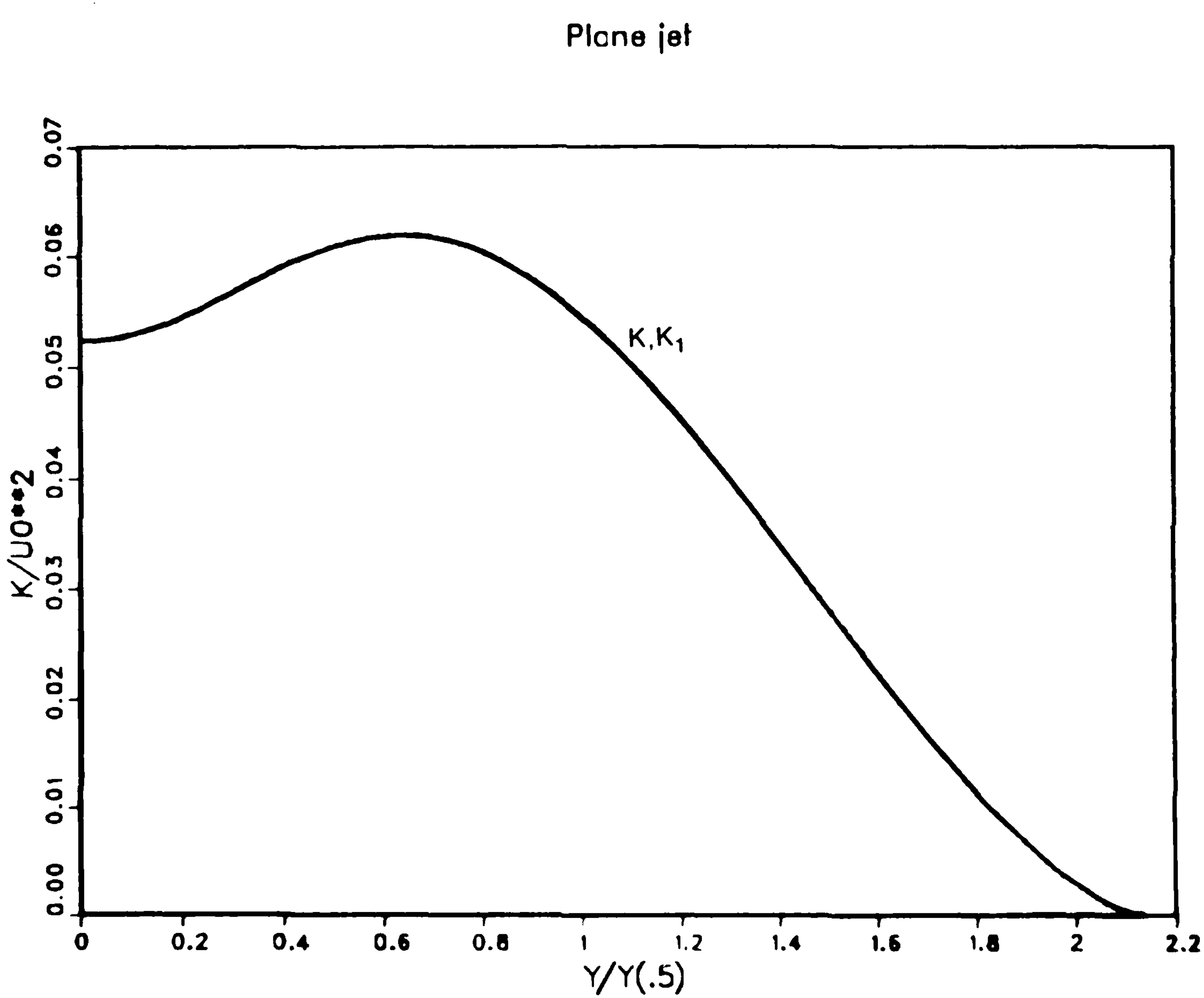

Fig. 13c Development of zonal kinetic energy
a) X/D = 10, b) X/D = 30, c) X/D = 60

Plane jet

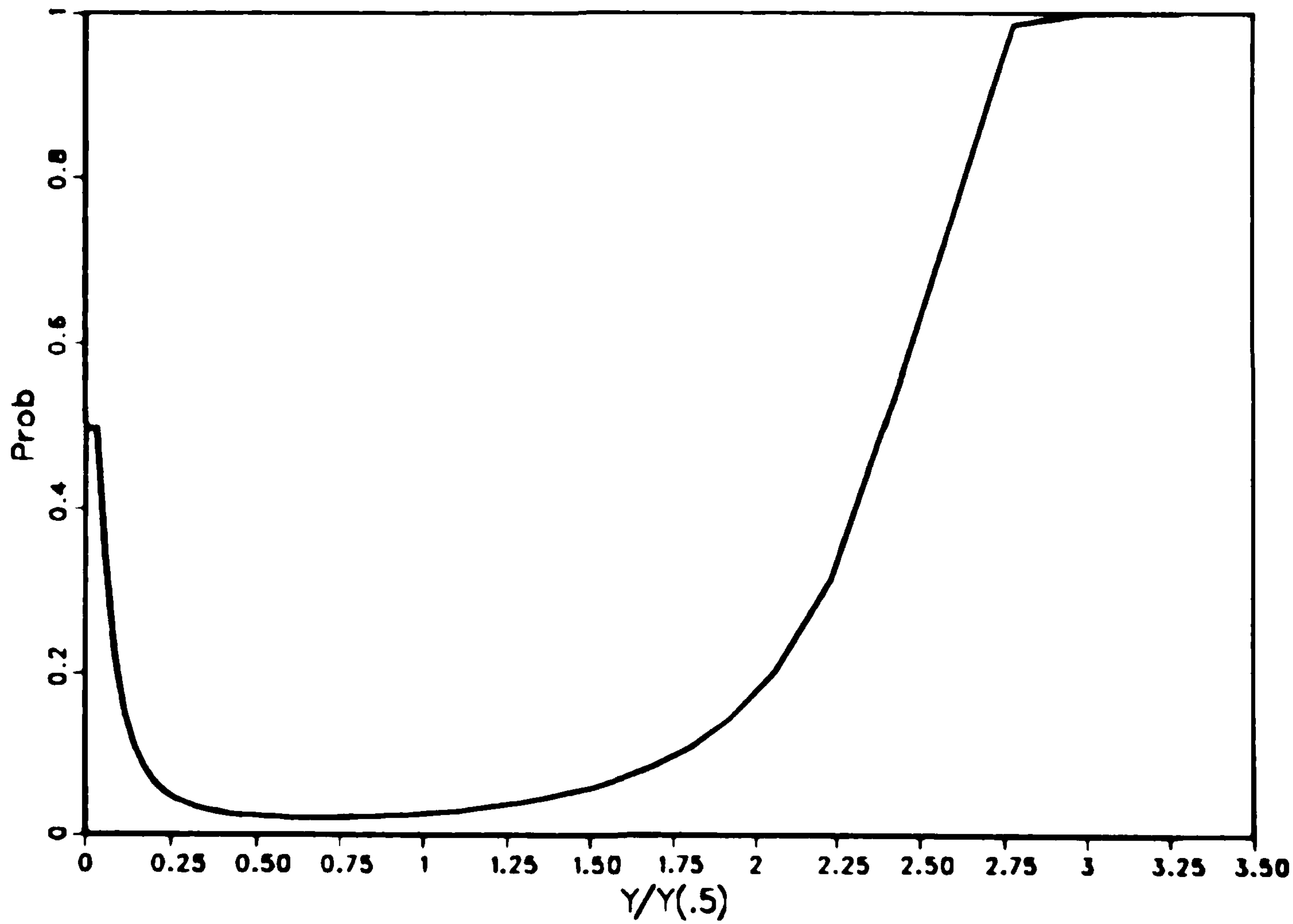

Fig.14a

Plane jet

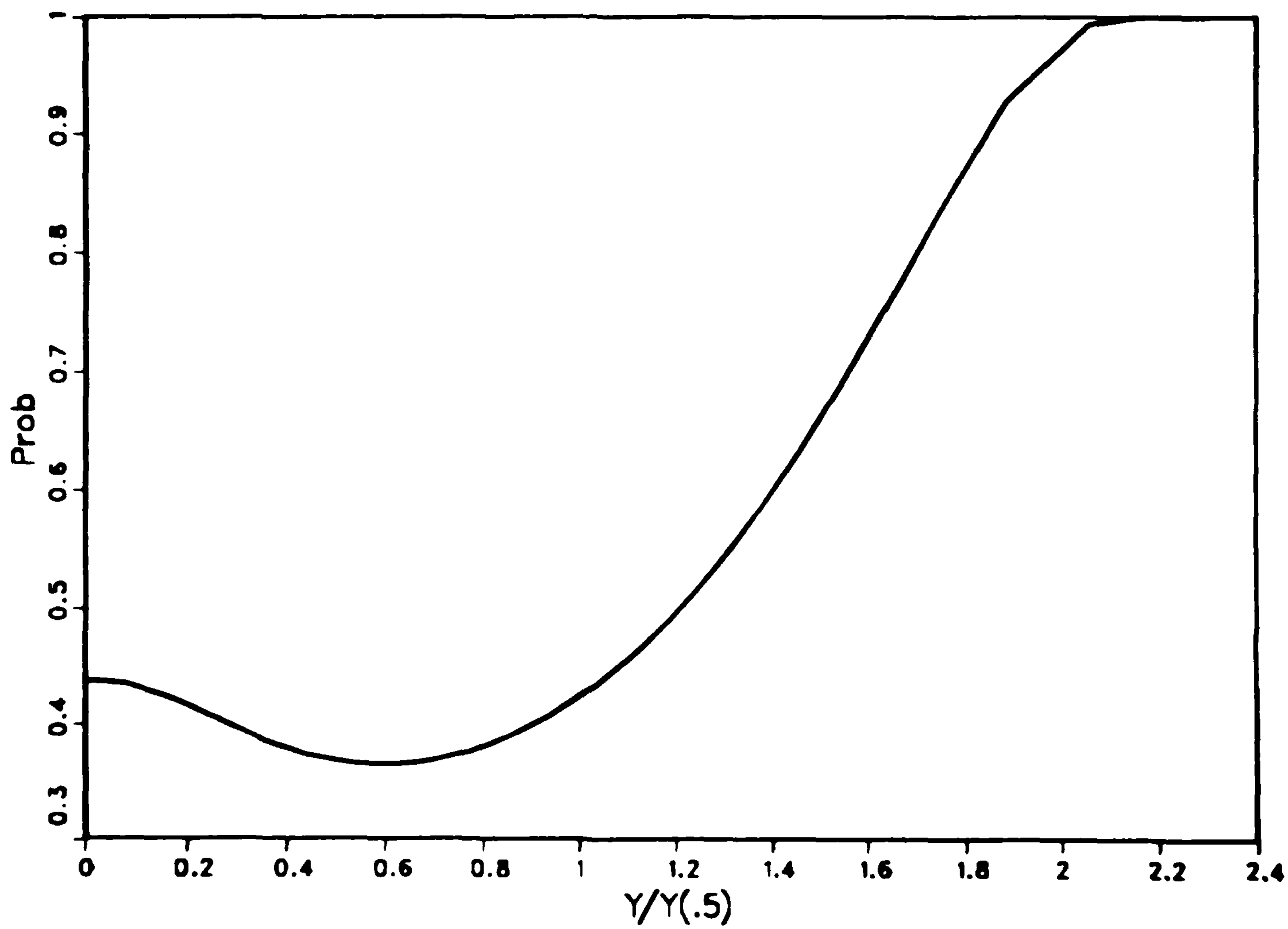

Fig.14b

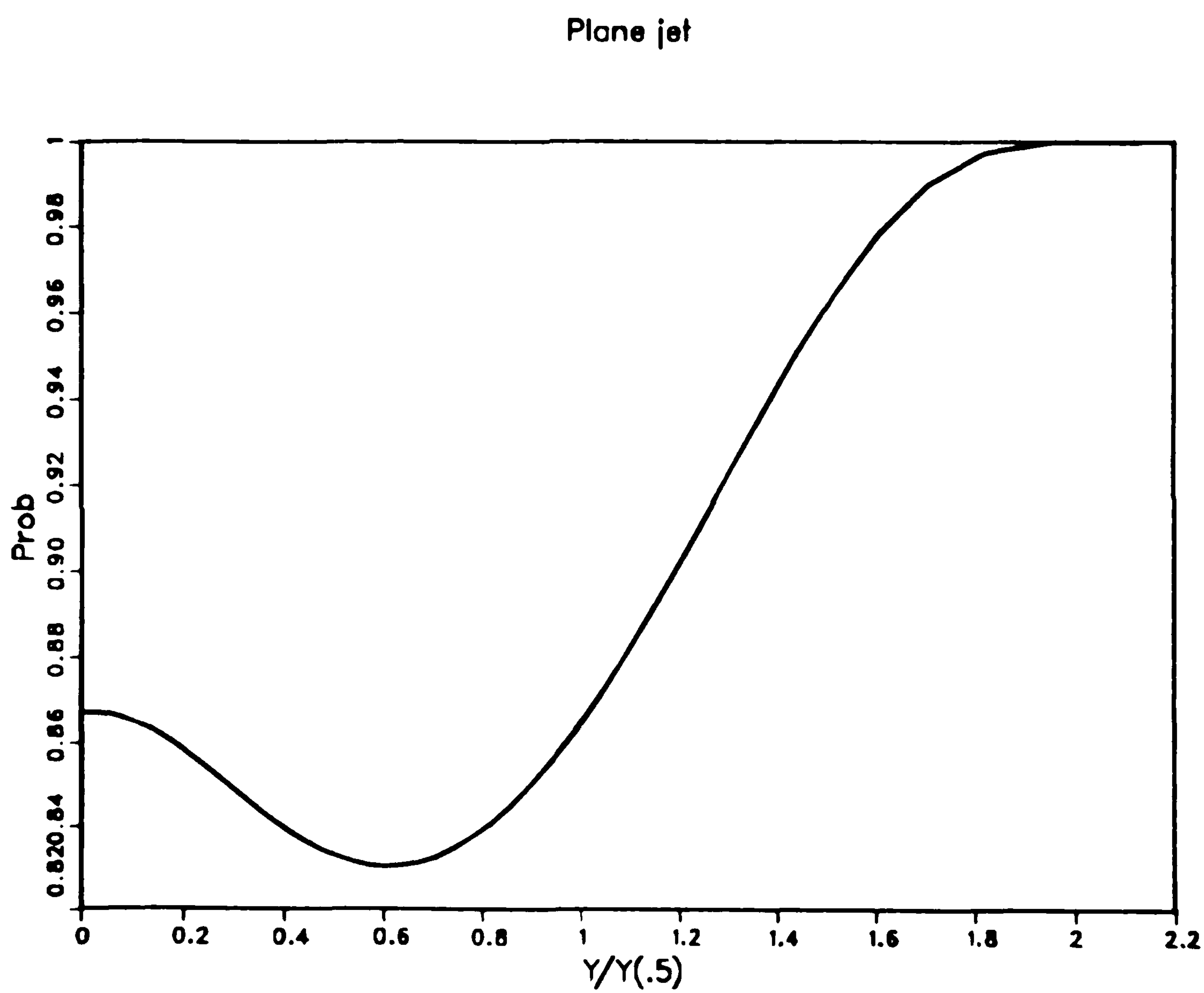

Fig. 14C Development of the probability for $\varepsilon < \varepsilon_i$
a) X/D = 10, b) X/D = 30, c) X/D = 60

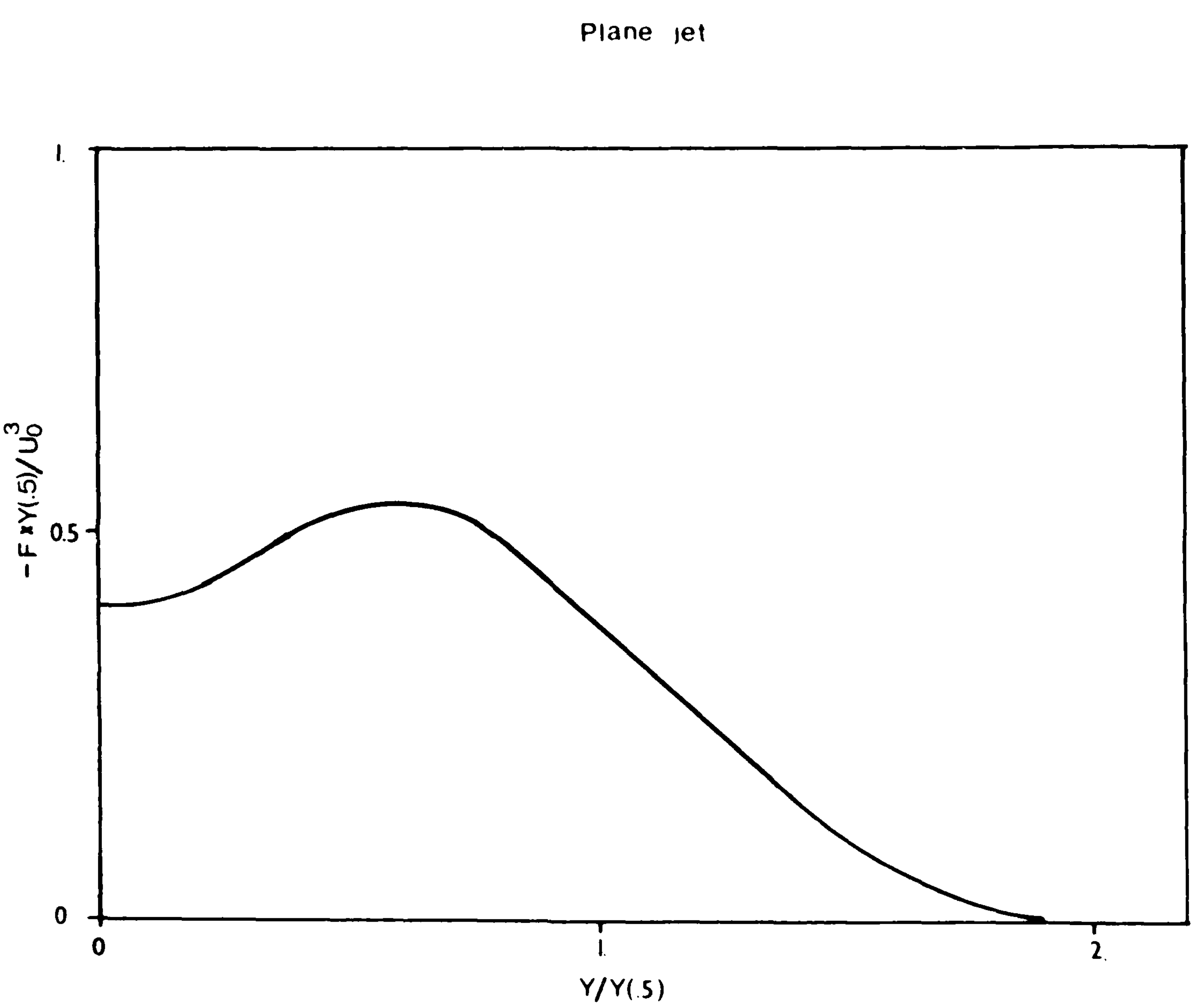

Fig. 15 The energy transfer term, $X/D = 60$

Plane jet

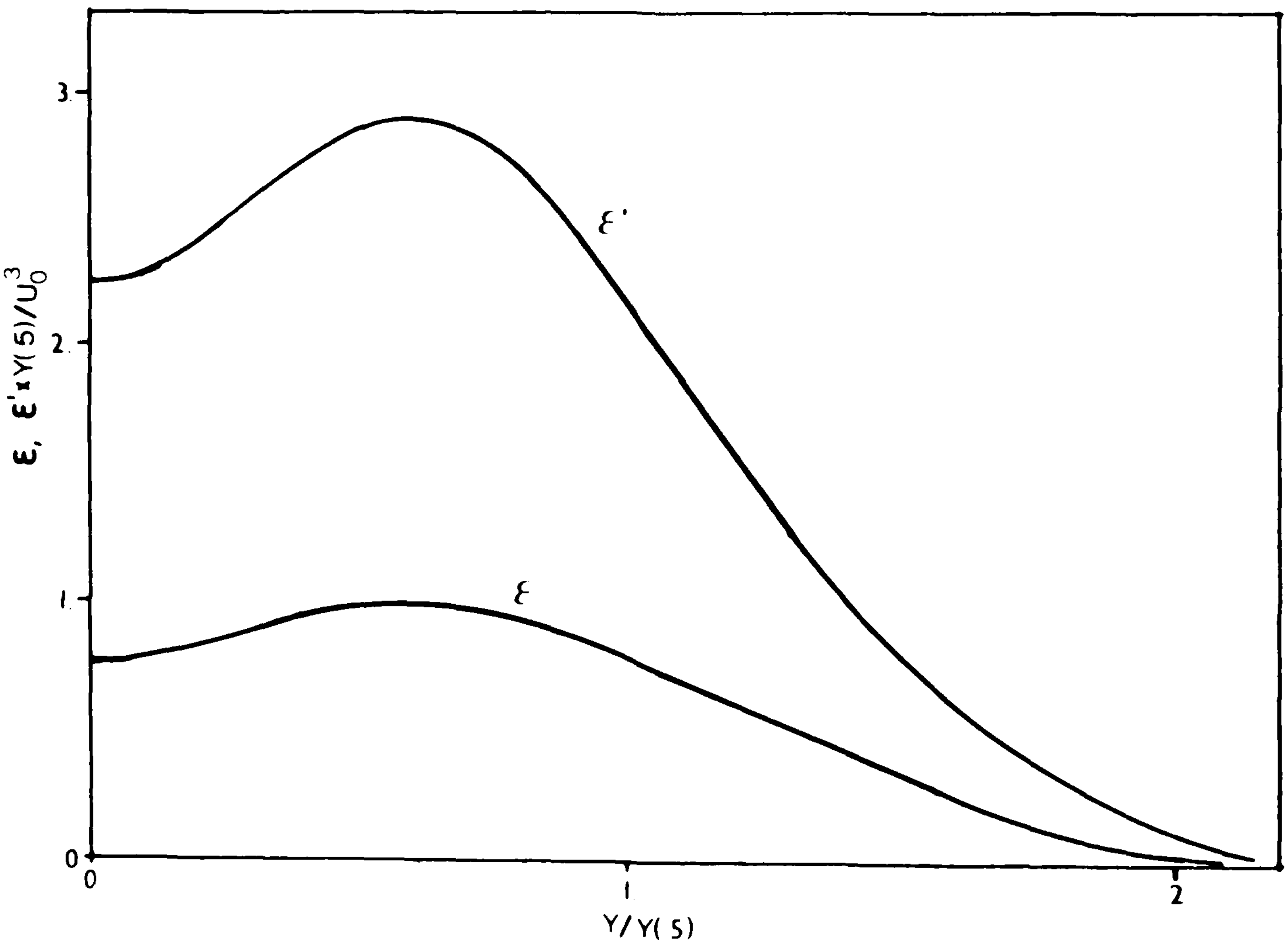

Fig. 1n Mean and standard deviation of ε , X/D = 60 .

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